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ON THE SPECTRUM OF IONIZED CALCIUM (*Ca* II)

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ABSTRACT

Series lines in the spectrum of ionized calcium.—Results are given of measurements of the spectrum of *Ca* II from various sources, showing the existence of a system of combination series of the same sort as occur in *Mg* II. The second and third members of the principal series have been found, and series of the types $2\pi - m\sigma$, $2\pi - m\delta$, and $3\phi - m\phi'$ detected, together with other combinations. *Tables* of all the series and combinations, and of the term-values and Rydberg denominators, are included.

Eleven years ago, Fowler[1] published his analysis of the spectrum of ionized *Mg*, and indicated the manner in which similar spectra should be attacked. The spectrum of ionized *Ca* (here referred to as the second *Ca*-spectrum, or *Ca* II) is arranged on the same plan as *Mg* II, and the main facts about this spectrum are to be found in the tables in Fowler's *Report on Series in Line Spectra*. The present paper presents additional results to complete these tables, and to give slightly more accurate values for some of the terms already known. The observations are the work of the senior author; the discussion, of both jointly.

The observations on this spectrum extend over a period of years, and are based on photographs taken in nearly all parts of the spectrum, and with a large variety of sources. In an earlier paper[2] it was

[1] *Philosophical Transactions*, A, **214**, 225, 1914.

[2] F. A. Saunders, *Astrophysical Journal*, **32**, 153, 1910.

noted that there was a curious complex "vacuum pair" in the *Ca* spectrum (λλ 5021, 5020, 5001) which, being a pair, appeared to belong to the spectrum of ionized *Ca*, and, from its structure, to be a member of a diffuse series. Several other pairs with the same separation ($\Delta\nu = 78$) exist, and we have found that they can all be accounted for as combinations between the terms of the known σ and δ series and a new double term which has just the magnitude and separation which might be anticipated for 2π.

Our recent observations were carried on with a Hilger quartz spectrograph, and a *Ca*-metal vacuum arc, with the image of the negative arc terminal thrown on the slit, precisely as Fowler has recommended. Long exposures then brought out the lines of this system. These observations were supplemented by a study of the plates used by Crew and McCauley in their work on the vacuum-arc spectrum of *Ca*[1] which were most kindly loaned to us by Professor Crew. Their excellent definition and high resolving power gave us better values for wave-lengths in many cases than we should otherwise have obtained, and the source they used, the rotating arc *in vacuo*, seems to be favorable to the production of this spectrum. We are also especially indebted to Professor T. Lyman, of Harvard University, for an opportunity of studying his *Ca* plates in the Schumann region, one of which yielded several interesting lines.

The new lines fall into two series, $2\pi - m\delta$ and $2\pi - m\sigma$ as expected, and in addition a pair appears which is $2\delta - 4\phi$; four lines which belong to the series $3\phi - m\phi'$, analogous to a series found by Fowler in *Mg* II; the series $1\delta - m\pi$ is established by finding a second pair belonging to it, and traces of a third; the two combination series $2\delta - m\phi$ and $2\sigma - 3\pi$ are represented by one pair each; and one line appears which seems to be the combination $1\pi_1 - 2\pi_1$. The principal series $1\sigma - m\pi$, which contains the famous H and K lines of the solar spectrum, has been extended by the discovery of the next pair (λλ 1649.96 and 1652.02, shown in Fig. 1) on one of Lyman's plates; and the third pair of this series (λ 1342.2, unresolved) shows on the same plate.

The adjustment of the term-values has been made as follows. Starting with the lowest term, 1σ, the *relative* values of 1π, and then

[1] *Astrophysical Journal*, **39**, 29, 1914.

those of 2σ, 3σ, 1δ, 2δ, 3δ, are fixed by well-observed lines. The combinations $1\delta - 2\pi$, $1\pi - 2\sigma$, and $2\pi - 3\delta$ then determine 2π and 3π, and the good lines $2\pi - m\sigma$ and $2\pi - m\delta$ determine the remaining terms of the sharp and diffuse sequences. The latter, thus extended, is long enough to give a reliable limit, and fix the term-values *absolutely* to about one unit in $\Delta\nu$.

The value of 4ϕ is found from the pair $2\delta - 4\phi$. The remaining ϕ-terms depend upon lines in the extreme ultra-violet. The value of 3ϕ thus found is confirmed by the limit of the series $3\phi - m\phi'$, which can be accurately found, and agrees within a unit or two. Those of 5ϕ and 6ϕ have been computed from a Ritz formula based on 3ϕ and 4ϕ.

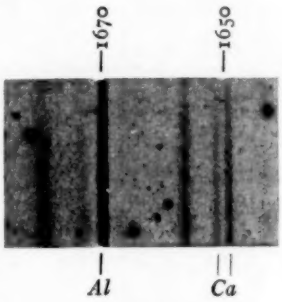

FIG. 1

The differences between the observed position of the lines and those computed with the foregoing term-values are given in Table I. The zero residuals correspond to the combinations used to determine the terms. Almost all the measured wave-lengths shorter than λ 1900 appear to be too great by about 0.5 A, but the diffuseness of the lines in question makes such an error not at all surprising.

Table II gives the term-values which have been adopted and also the corresponding Rydberg "denominators." The differences of these last quantities from integral values are the "residuals" or "quantum defects" for the various series. On plotting them against the term-values, straight lines are obtained in all cases, which indicates that all the series can be very closely represented by formulae of the Ritz type. The differences (Δ) of the residuals for the components of the successive π-terms are almost equal *inter se*, and the same is true for the δ-terms.

For the ϕ-terms, only two good values are available, and the other two are computed from the formula. They are not far from the values which would follow from the observed lines if the systematic error mentioned above were corrected, and are doubtless preferable to these.

It may be noted in passing that by far the easiest way to find

TABLE I

Ca+ DOUBLETS

	Int.	ν	$\Delta\nu$	m	O.−C. $\Delta\nu$	O.−C. $\Delta\lambda$
	First Principal Series. $1\sigma - m\pi$					
λ						
3933.664	(10 R)	25414.41	222.85	(1)	0.0	0.00
3968.465	(9 R)	25191.56			0.0	0.00
λ Vac.						
1649.96	(2)	60607.5	75.5	(2)	− 5.5	+0.15
1652.02	(1)	60532.0			− 2.6	+0.07
1342.07	(1)	74512	.....	(3)	− 9.7	+0.18
					+ 2.6	−0.47
	First Sharp Series. $1\pi - m\sigma$					
λ						
−3933.664	(10 R)	−25414.41	222.85	(1)	0.0	0.00
−3968.465	(9 R)	−25191.56			0.0	0.00
3736.903	(9)	26752.55	222.91	(2)	0.0	0.00
3706.022	(8)	26975.46			0.0	0.00
2208.606	(3)	45263.26	222.71	(3)	0.0	0.00
2197.791	(2)	45485.97			− 0.1	0.00
λ Vac.						
1851.10	(2)	54021.9	220	(4)	−13.4	+0.46
1843.6	(1)	54242			−16	+0.5
1698.9	(1)	58861	226	(5)	−27	+0.8
1692.4	(1−)	59087			−24	+0.7
	First Diffuse Series. $1\pi - m\delta$					
λ						
−8498.00	(8)	−11764.25	60.81	(1)	0.0	0.00
−8542.15	(10)	−11703.44	222.89		.0	.00
−8662.11	(9)	−11541.36			0.0	0.00
3181.283	(6)	31424.80	19.19	(2)	− 0.1	+0.01
3179.340	(10)	31443.99	222.89		. .0	.00
3158.877	(9)	31647.69			0.0	0.00
2113.19	(1)	47307.3	9.04	(3)	− 0.3	+0.01
2112.763	(2)	47316.34	223.28		.0	.00
2103.239	(2)	47530.58			+ 0.2	−0.01
λ Vac						
1815.04	(1)	55095.2	222.5	(4)	− 18.4	+0.61
1807.74	(1)	55317.7			− 14.7	+0.48
1680.5	(1)	59506	227	(5)	−18	+0.5
1674.1	(1)	59733			−14	+0.4
	First Fundamental Series. $1\delta - m\phi$					
λ Vac.						
1840.21	.....	54341.6	63.0	(3)	− 1.4	+0.05
1838.08	.....	54404.6			+ 0.8	−0.03
1555.1	.....	64304	66	(4)	−19	+0.5
1553.5	.....	64370			−14	+0.3
1434.3	.....	69720	58	(5)	−27	+0.6
1433.1	.....	69778			−30	+0.6
1370.6	.....	72960	80	(6)	−55	+1.0
1369.1	.....	73040			−36	+0.7
	G-Type Series. $3\phi - m\phi'$					
λ						
(Not obs.)	.....	(10109±)	.....	(4)	.....	.....
6456.91	.....	15483.0	.....	(5)	0.0	0.00
5339.29	.....	18723.9	.....	(6)	0.0	.00
4800.16	.....	20826.8	.....	(7)	0.0	.00
4489.22	.....	22269.4	.....	(8)	0.0	0.00

TABLE I—*Continued*

	Int.	ν	$\Delta\nu$	m	O.−C. $\Delta\nu$	O.−C. $\Delta\lambda$
	Second Principal Series. $2\sigma-m\pi$					
λ						
4479.29	(1)	22354.7	35.9	(3)	0.0	0.00
4472.09	(0)	22318.8			0.0	0.00
	Second Sharp Series. $2\pi-m\sigma$					
λ						
(Not obs.)		(10065)		(3)		
5307.30		18836.7	78.3	(4)	0.0	0.00
5285.34		18915.0			−0.1	+0.03
4220.13		23689.3	78.4	(5)	0.0	0.00
4206.21		23767.7			0.0	0.00
	Second Diffuse Series. $2\pi-m\delta$					
λ						
(Not obs.)		(12118)		(3)		
5021.14	(1)	19910.2	4.8	(4)	0.0	0.00
5019.98	(3)	19915.0	78.4		.0	.00
5001.49	(2)	19988.6			0.0	0.00
4110.33	(0)	24322.2	3.0	(5)	0.0	0.00
4109.83	(1)	24325.2	78.5		.0	.00
4097.12	(1)	24400.7			+0.1	−0.02
3694.31	(0)	27061.0	1.6	(6)	+0.2	−0.03
3694.11	(1−)	27062.6	77.9		.0	.00
3683.71	(1−)	27138.9			−0.3	+0.04
3461.90	(1−)	28877.8	77.1	(7)	0.0	0.00
3452.67	(1−)	28954.9			0.0	0.00
3316.58	(0)	30142.9	77.4	(8)	0.0	0.00
3308.08	(0)	30220.3			0.0	0.00
	Second Fundamental Series. $2\delta-m\phi$					
λ						
(Not obs.)		11196 11215		(3)		
4722.58		21175.3	19.4	(4)	−0.2	+0.04
4718.16		21194.7			+0.1	−0.02
	Combination Series. $1\delta-m\pi$					
λ						
8498.00	(8)	11764.25	60.81	(1)	0.0	0.00
8542.15	(10)	11703.44	222.89		.0	.00
8662.11	(9)	11541.36			0.0	0.00
2132.25	(1)	46884.2	17.9	(2)	−0.2	+0.01
2131.43	(2)	46902.1			+0.1	0.00
(Not obs.)		(46963.0)				
λ Vac.						
1644.25	(0)	60818	Blend	(3)	+7 −18	−0.19 +0.49
	Other Combinations. $1\pi_1-2\pi_1$					
λ						
2840.16		35199.0			+0.4	−0.03

out whether the lines of a series can be represented by a Ritz formula, and to obtain its constants if they do, is to assume a plausible value for the limit, compute the terms and the "residuals," and plot the

TABLE II

BEST VALUES OF TERMS. $Ca+$

m	$m\sigma$	$m\pi$	$\Delta\nu$	$m\delta$	$\Delta\nu$	$m\phi$	$m\phi'$
1	95748.0	{70333.6 / 70556.4}	222.8	{82037.0 / 82097.8}	60.8	.....	.....
2	43581.0	{35135.0 / 35213.4}	78.4	{38889.6 / 38908.7}	19.1	.....	.....
3	25070.3	{21226.3 / 21262.2}	35.9	{23017.3 / 23026.0}	8.7	27694.0	.....
4	16298.3	.....	.....	{15220.0 / 15224.8}	4.8	17714.1	[17585]
5	11445.7	.....	.....	{10809.8 / 10812.8}	3.0	12290	12211.0
6	.....	.....	.....	{8072.4 / 8074.2}	1.8	9022	8970.1
7	.....	.....	.....	{6257.2 / 6258.5}	1.3	.....	6867.2
8	.....	.....	.....	{4992.1 / 4993.1}	1.0	.....	5424.6

TABLE III

RYDBERG DENOMINATORS

m	$m\sigma$	$m\pi$	Δ	$m\delta$	Δ	$m\phi$	$m\phi'$
1	2.1406	2.4975	0.00392	2.3126	0.00084	.....	.....
2	3.1728	3.5337	0.00394	3.3587	.00076	.....	.....
3	4.1832	4.5463	0.00384	4.3658	.00082	3.9801	.....
4	5.1882	.....	.....	5.3689	.00084	4.9766	[4.9949]
5	6.1912	.....	.....	6.3706	.00088	5.9747	5.9941
6	.....	.....	.....	7.3721	.00082	6.9736	6.9934
7	.....	.....	.....	8.3732	.00086	.....	7.9928
8	.....	.....	.....	9.3745	0.00092	.....	8.9928

latter against the terms themselves (or the wave-numbers of the original lines). The value of the limit which gives a rectilinear plot can then be found by trial and error, in a remarkably short time—if indeed the series is Ritzian at all.

The Ca^+ spectrum—like others of "stripped atoms" having but

one available electron—is remarkably regular, the only notable peculiarity being the presence of the low, metastable term 1δ.

In none of the series, however, have we been able to find as many lines as might be hoped for. The difficulty seems to be that when the *Ca*-atoms are violently disturbed, they are not strongly concentrated in the form Ca^+, but pass over in considerable numbers to the doubly ionized condition, Ca^{++}, which of course produces the spectrum *Ca* III instead of the one we sought. We have measured a hundred lines, or so, of the latter spectrum, in almost complete agreement with the lists recently published by Anderson.[1]

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[1] *Astrophysical Journal*, **59**, 76, 1924.

ON SOME PROBLEMS CONNECTED WITH THE LUMINOSITY LAW

By WILLEM J. LUYTEN

ABSTRACT

Derivation of the luminosity law.—The most valuable material at present available for the determination of the luminosity-curve for all stars in space consists of systematic surveys of proper-motion stars. When the frequency function of the reduced proper motions is known, and the correlation between absolute magnitude and reduced proper motion is calculated by means of trigonometric parallaxes, a first approximate expression for the luminosity law may be found from the combination of these two. The present material leads to the tentative conclusion that the luminosity-curve, at least for the portion which is involved in the discussion, may be represented by a normal error-curve with a maximum between absolute magnitude 8 and 9, and a standard deviation of not less than ± 2.5.

Comparison of results derived from selected groups of stars with predictions made on the basis of Kapteyn's luminosity-curve.—Analysis of the data collected for the stars nearer than 25 parsecs shows that in the solar neighborhood there is an excess of absolutely bright stars over the number predicted by Kapteyn. It also appears that there is no marked gap between K giants and K dwarfs, but that the change is continuous. Our present views on the absolute magnitudes of stars of given spectral class, combined with the known numbers of stars brighter than apparent magnitude 8.25 belonging to each spectral subdivision, lead to the conclusion that Kapteyn's luminosity and density laws predict too many supergiants and not enough ordinary giants. Evidence derived from the galactic concentration of stars of different spectral class renders improbable the existence of a universal luminosity-curve, as it appears that the percentage density of A stars is much higher in low galactic latitudes than near the poles of the galaxy.

Repeated discussion of the available star-counts, parallax and proper-motion material, led Kapteyn[1] to adopt a normal Laplacean error-curve for the function representing the distribution of stellar luminosities and for that representing the change in density with increasing distance from the sun. Throughout the entire range of observations, which seemed to extend well beyond the maximum of the curve, the analytical luminosity-curve accorded very well with the empirical data.

Since then it has been pointed out by various investigators[2] that certain selected groups of stars showed a distribution of luminosities deviating more from that predicted by Kapteyn's laws than would

[1] *Contributions of the Mount Wilson Observatory*, No. 188, 1920; *Astrophysical Journal*, **52**, 23, 1920.

[2] Hertzsprung, *Bulletin of the Astronomical Institutes of the Netherlands*, **1**, 21, 1922; Kienle, *Astronomische Nachrichten*, **218**, 119, 1922; Öpik, *Publications de l'Observatoire Astron.*, Tartu, **25**, 2, 1922; Luyten, *Lick Observatory Bulletin*, **11**, 24, 1922.

a priori have been expected. While awaiting further observational data, in particular accurate knowledge of the numbers and mean distances of stars of large proper-motion, which will render a revision of the luminosity law possible, it may be of value to analyze again the material at our disposal. In addition, the possibility of a universal luminosity-curve may be tested by using the analytical form adopted by Kapteyn as a basis for making crucial predictions which are capable of accurate checking by means of direct observations.

1. THE DERIVATION OF THE LUMINOSITY LAW

The observational material consists chiefly of large proper motions, but these cannot be made use of statistically without making certain auxiliary assumptions. In the present discussion we shall select our observational material from the proper-motion surveys of Wolf.[1] We shall assume that the lists published by him contain, within limited areas of the sky, all stars whose proper motions are larger than $0''.2$, and whose apparent magnitudes are brighter than 12.5. We may, at least for the present, safely ignore the difference between Wolf's relative proper motions and their absolute values. Furthermore, we shall assume that Wolf's magnitudes may be reduced to the Harvard visual standard scale by adding 0.5 to his values.[2] The subsequent line of attack may then briefly be described as follows.

For the stars under consideration, the frequency-curve of the reduced proper motions, $F(H)$, can be formed. The available measured parallaxes enable us to determine the mean absolute magnitude also as a function of H, $G(H)$. A combination of the two relations will transform F into a frequency-curve of absolute magnitude $\phi(M)$, and from ϕ we then deduce the probable form of $\Phi(M)$, the luminosity-curve of all stars in space. In this final step we must take into account the selection entering into $G(H)$, and the difference between $F(H)$ and the corresponding curve for all stars in space.

As a first step toward the derivation of $G(H)$, all modern

[1] *Veröff. Bad. Sternw. Heidelberg*, **7**, No. 10, 1919.

[2] Cf. *Groningen Publications*, No. 30, p. 15, Table I, 1920.

trigonometric parallaxes of stars with proper motions in excess of 0".2 have been collected.[1] A plot of the absolute magnitudes of these stars against H is given in Figure 1. The range in H is from 2.6 (α Tauri) to 19.8 (Barnard's star); that in M from -2.4 (δ Corvi)

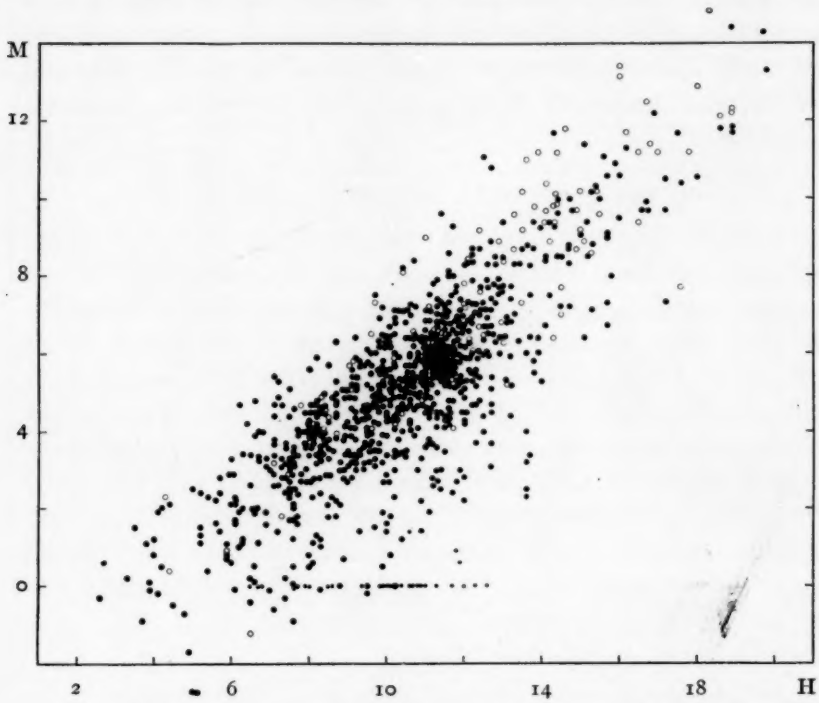


FIG. 1.—Relation between absolute magnitude, $M=m+5+5 \log \pi$, and reduced proper motion, $H=m+5+5 \log \mu$, for stars with proper motions exceeding 0".2 annually. Open circles represent fainter components of double stars; small dots represent negative or very uncertain parallaxes.

to 14.4 (van Maanen's star). The dots are widely scattered, but it is obvious that for the brighter stars a large part of this scattering (below and to the right of the bulk of the dots) is due to negative errors in the measured parallaxes. Some stars, however, do deviate very largely from the average; for example, 16c Ursae Majoris, $H=2.7$, $M=$ about 4; and RZ Cephei, $H=11-12$, $M]$ 0,[2] but the assumption that such extremes are very scarce seems well justified.

[1] Cf. Schlesinger, *Catalogue of Parallaxes*, 1924.

[2] *Harvard Bulletin*, No. 802, 1924, and No. 773, 1922.

The determination of the regression-curve of M on H meets with difficulties, principally in the evaluation of the negative parallaxes. This might be overcome by taking median values of M in small intervals of H, or by taking M corresponding to the median value of the reduced parallaxes.[1] As it was found that the difference in the results of either of these procedures from the result obtained by simply rejecting the stars in question is very small, the last method has been arbitrarily adopted. Furthermore, those stars which are fainter companions to bright stars were also rejected, leaving a total of 782 stars. The correlation coefficient r, and the regression line $M=a+cH$, were determined, with the result:

$$r=0.81\pm0.014$$
$$a=-2.9$$
$$c=+0.76$$

The agreement between this mathematically computed regression line and the points which form the empirical regression-curve is shown in Figure 2. The middle point of each line represents the mean M taken over equidistant intervals of H; the length of the vertical line equals twice the standard deviation of this mean. The accordance is as close as might be expected, but the systematic character of the deviations suggests that a quadratic of the form $M^2-MH+(A-B)M+BH-(AB+C)=0$, being a hyperbola whose asymptotes make an angle of 135°, would give a better fit.[2] One might also use a parabola of the form $M=a+cH+dH^2$, with d about 0.017. Owing to the selection for large angular motion operative for the absolutely faint stars, this value of d must be regarded as a minimum.

For the present we may, in first approximation, adopt a straight regression line for M on H for all values of H between 6 and 18. We may not, however, use this linear relation directly to convert the H of the Wolf stars into M, but must take into account the

[1] Cf. *Lick Observatory Bulletin*, **10**, 135, 1922; *Proceedings of the National Academy of Sciences*, **9**, 317, 1923.

[2] A good fit may be obtained by using $A=6.34$, $B=-1.04$, $C=4.78$; the hyperbola determined by these constants is shown in Fig. 2 by a dotted line. The meaning of the constants is immediately clear: A indicates the highest velocity, B the brightest absolute magnitude on the regression-curve.

differences in the selection between these two groups of stars. Owing to the preference given by parallax observers to (*a*) apparently bright stars and (*b*) stars with large proper motion, the representation of the bright stars with proper motions larger than 0".2 is very nearly complete; whereas very nearly all stars fainter than the tenth apparent magnitude included in the parallax lists have proper motions of 0".4 or larger. With the well-known statistical increase in linear speed with decrease in absolute brightness, it would seem at first sight as if our relation $M = -2.9 + 0.76\ H$ would give too faint luminosities. A little closer consideration, however, shows that this is not the case, and that rather the reverse should be expected. The paradox, that by selecting the fainter stars we get too bright an absolute magnitude, finds its origin in the fact that we are dealing with a regression-curve. To take a specific example, let us consider those stars whose H equals 14.0. Among those included in the parallax lists we find an unduly large proportion with very large proper motion, such as stars of magnitude 9 and annual motion of 1". On the other hand, stars of magnitude 12 and a proper motion of 0".25 occur less frequently in our lists. If, on the average, the former group of stars has a linear velocity of 95 km/sec., and the latter group a velocity of 47 km/sec., then the absolute magnitudes would be 7.5 and 9.0, respectively. Consequently, the mean absolute magnitude indicated by the regression line based on the measured parallaxes would be too bright. Of course, extreme cases like these will

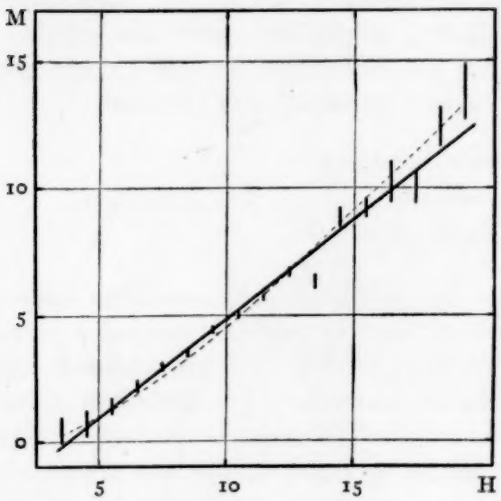


FIG. 2.—Comparison between computed and empirically derived points constituting the regression line M on H for stars with proper motions exceeding 0".2 annually.

not occur in practice, but nevertheless the tendency of giving too bright an absolute magnitude will persistently be found in the regression-curves.

The actual correction to be applied to our regression-curve to reduce it to the *M-H* relation for *all* stars brighter than magnitude 13 with proper motions larger than 0".2 annually will probably be very small, and may perhaps even be neglected, in view of the uncertainties introduced by our arbitrary exclusion of negative

TABLE I

MEAN ABSOLUTE MAGNITUDES OF STARS WITH PROPER MOTIONS EXCEEDING 0".2 ANNUALLY, AS A FUNCTION OF *H*

H	All Stars	*m*]9·1	*m*]8·1	*m*]7·1	*m*]6·1
3·55	0.5	0.5	0.5	0.5	0.5
4·55	0.7	0.7	0.7	0.7	0.7
5·55	1.4	1.4	1.4	1.4	1.4
6·55	2.2	2.2	2.2	2.2	2.2
7·55	3.0	3.0	3.0	3.0	2.8
8·55	3·5	3·5	3·5	3·5	3.6
9·55	4·4	4·4	4·5	4·5	4·4
10·55	5.0	5.0	5.0	4·9	5.0
11·55	5·7	5·5	5·3	5.1	5.0
12·55	6.7	6.7	6.4	6.6	6.6
13·55	6.3	6.2	6.7	6.4	6.4
14·55	8.8	8.4	8.2	8.2	8.0
15·55	9.2	9.4	8.6	6.7	.....
16·55	10.2	9.7	9.7	.....	.....
17·55	10.0	10.0	.....	.....	.....

parallaxes. An empirical estimate of the correction may be obtained by determining the regression-curve anew after rejecting for successive approximations all stars fainter than magnitude 9, those fainter than magnitude 8, etc. Table I shows the mean absolute magnitudes for the different groups in successive intervals of *H*. The heavy lines through the table indicate the portion where the standard deviations of the mean values given are less than ±0.4. The change toward brighter absolute magnitudes, when we drop the apparently fainter stars is only slight but in the direction demanded by the paradox.

Tentatively we therefore adopt the relation $M = -2.9 + 0.78\,H$ as representing the desired regression line for stars brighter

than the twelfth magnitude with proper motions exceeding 0".2 annually.

For the derivation of the luminosity law from the Wolf stars we shall first have to determine the function $F(H)$. It has been shown that this curve must be skew and of a "beaky" form.[1] Owing to the uncertainties in our material, and judging from the shape of the curve actually found, it seems allowable to represent $F(H)$ by a normal error-curve with a mean value of 14.3, and a standard deviation of ± 2.1. The agreement between the curve and the observations, plotted on arithmetic-probability paper is shown in Figure 3. The only remaining unknown quantity is the dispersion in absolute magnitude for stars of given H. This may be derived from the data shown in Figure 1. Assuming, as has been done tacitly in the computation of r and the regression line, that the distributions in M and H follow a normal-error law, we find the standard deviation of the M-frequency-curves for stars of given H to be $\sigma_M = \pm 1.41$. Taking the actual frequency-curves of M in the six intervals of H: 7.0–7.9, ... 12.0–12.9, we find $\sigma_M = \pm 1.53$. For further calculations we shall assume $\sigma_M = \pm 1.5$, hoping that the various systematic sources of error, entering in its determination, more or less counterbalance one another.

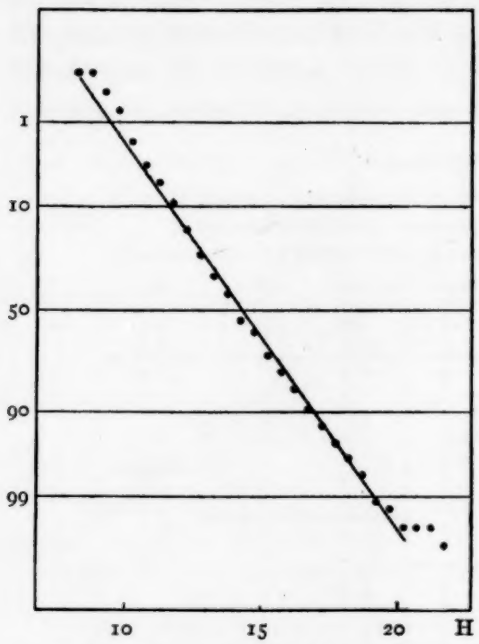


FIG. 3.—Frequency-curve of reduced proper motions of Wolf stars plotted on arithmetic-probability paper. Ordinates represent percentages of stars for which H is less than the values indicated by the corresponding abscissae.

[1] *Proceedings of the National Academy of Sciences*, 10, 260, 1924.

The derivation of $\phi(M)$ is now a simple matter. Writing $F(H)dH=\frac{l}{\sqrt{\pi}}e^{-l^2(H-H_0)^2}dH$, $M=a+cH$, we have immediately:

$$\left.\begin{aligned}\phi(M)dM&=\frac{k}{\sqrt{\pi}}e^{-k^2(M-M_a)^2}dM\\ M_a&=a+cH_0=8.2\\ \frac{1}{k^2}&=\frac{c^2}{l^2}+2\sigma_M^2\,,\qquad k=0.32\end{aligned}\right\}\tag{2}$$

The final step toward the solution of our problem consists in the determination of the difference between $\phi(M)$ and $\Phi(M)$, the luminosity-curve for *all* stars in space. This reduction again involves many assumptions, and a numerical solution can be given only after first assuming the functional form and the numerical values of the constants of $\Phi(M)$ itself.

A numerical solution might be found in the following way. Let $\Phi(M)$ be the luminosity law, $\Psi(T)$ the distribution law for the linear tangential velocities, and $\Delta(r)$ the density law; then the number of stars (dN) whose distances lie between r and $r+dr$, whose absolute magnitudes are between M and $M+dM$, and whose tangential velocities lie between T and $T+dT$, is given by

$$dN=4\pi r^2\cdot\Delta(r)dr\cdot\Phi(M)dM\cdot\Psi(T)dT\,,$$

and the total number of stars brighter than apparent magnitude m_c and with annual proper motion larger than μ_c is given by

$$N(m_c,\mu_c)=\int_{-\infty}^{+\infty}dM\int_0^{\rho}dr\int_{\mu_c r}^{+\infty}dT[4\pi r^2\Delta(r)\cdot\Phi(M)\cdot\Psi(T)]\tag{3}$$

in which $\log\rho=0.2(m_c-M+5)$.

The mean absolute magnitude of these stars is given by

$$M(m_c,\mu_c)=\iiint MdN/N\,,$$

the limits of the triple integral being the same as in (3). We are dealing with faint proper-motion stars, accordingly with nearby

stars, and we may well assume that $\Delta(r)=$ constant. Further, assuming

$$\Phi(M)dM=\frac{h}{\sqrt{\pi}}e^{-h^2(M-M_0)^2}dM$$

$$\Psi(T)dT=\frac{t\ \text{Mod.}}{\sqrt{\pi}}e^{-t^2(\log T-a-bM)^2}dT/T\ ,$$

N reduces to

$$N=\text{Const.}\int_{-\infty}^{+\infty}e^{-L^2}dL\int_{-\infty}^{A-BL}e^{3u}du\int_{Cu-DL-E}^{+\infty}e^{-x^2}dx\ ,$$

in which

$$L=h(M-M_0)$$
$$x=t(\log T-A-CM)$$
$$u=\log\text{ nat. }r$$
$$A=(m_c-M_0+5)/5\ \text{Mod.}$$

$$B=1/5h\ \text{Mod.}$$
$$C=t\ \text{Mod.}$$
$$D=tb/h$$
$$E=t(a+bM_0-\log\mu_c)$$

Being unable to express either $N(m_c,\mu_c)$ or $M(m_c,\mu_c)$ as an explicit function of h, M_0, t, a, b, m_c, and μ_c, we have to be satisfied with taking the best numerical values for these constants and evaluating the integral numerically. Using Kapteyn's luminosity-curve, we have $h=0.28$, $M_0=7.69$; for simplicity's sake we have then taken $m_c=13.7$. Furthermore, we have adopted $a=0.54$, $b=0.056$, $t=2.8$, $\mu_c=0''.2$. Numerical quadrature then yields

$$M(m_c,\mu_c)=M_0-0.5\ .$$

If we should drop the restriction of apparent magnitude, and thus ask for the mean absolute magnitude of *all* stars whose proper motions exceed $0''.2$ annually, we would be able to integrate rigorously and obtain

$$M(>0''.2)=M_0+3b/2\ \text{Mod. }h^2=M_0+2.5\ .$$

If all our assumptions were correct we should have obtained $M(13.2, 0''.2) = M_0 - 0.5 = 7.2$. Instead we find from equation (2) that $M_a = 8.2$. Out of our five theoretical constants h, M_0, a, b, and t, the last three have entered more or less in our regression-curve, and the disagreement between the two values of $M(m_c,\mu_c)$ cited here is therefore in all probability due either to an erroneous assumption concerning the functional form of $\Phi(M)$ or $\Psi(T)$, or to an erroneous value of h or, in particular, of M_0. This last alternative seems at present preferable, and we conclude that the maximum of the luminosity-curve for all stars in space may be at an absolute magnitude somewhere between 8 and 9. This may be compared with the values 9.0 and 8.5 derived before.[1] It cannot be too strongly emphasized that the statement, "The luminosity-curve is an error-curve with its maximum somewhere between absolute magnitude 8 and 9," is not meant to convey any more than that the numbers of stars in space, absolutely brighter than this maximum, and nearer than, say, 50 or 100 parsecs, fit this curve rather closely. Of the numbers of stars in space absolutely fainter than +10 we know naught, and therefore it seems better for the present to let the functional form of the luminosity-curve remain unaltered, especially as the part beyond the maximum is of little importance in most statistical investigations. The exact form of the luminosity-curve, provided there is such a universal luminosity-curve, can only be determined after we have accumulated much more material on large proper motions and parallaxes than is now in our possession. That the ultimate form of the luminosity law may deviate considerably from a normal error-curve is not at all unlikely; in fact, Öpik has shown that the error-curve shape disappears if, instead of visual absolute magnitudes, bolometric or "true" absolute magnitudes are used.[2]

2. EVIDENCE DEDUCED FROM NEARBY STARS

At the meeting of the American Astronomical Society in December, 1923,[3] the writer reported some results derived from a discussion of stars nearer than 25 parsecs. Inasmuch as these results bear

[1] *Lick Observatory Bulletin*, **11**, 24, 1922.

[2] *Publications de l'Observatoire Astron.*, Tartu, **25**, 2, p. 23, 1922.

[3] *Popular Astronomy*, **32**, 224, 1924.

on the more general problem discussed here, a more detailed description than that presented at the meeting may add to the knowledge of the luminosity-curve.

According to our present views on stellar density, we know about 50 per cent of the stars nearer than 10 parsecs. When we consider stars out to 25 parsecs, the situation is far less favorable. Kapteyn's density law would place about three thousand stars within this distance. Of stars with well-established trigonometric parallaxes in excess of 0".039, we have only a few hundred. Additional stars are found by using spectroscopic, dynamical, moving-cluster, spectral and proper-motion parallaxes. The total number of stars thus available is 803, double stars counted as one. For the present, we have considered only those properties which bear on the luminosity-curve.

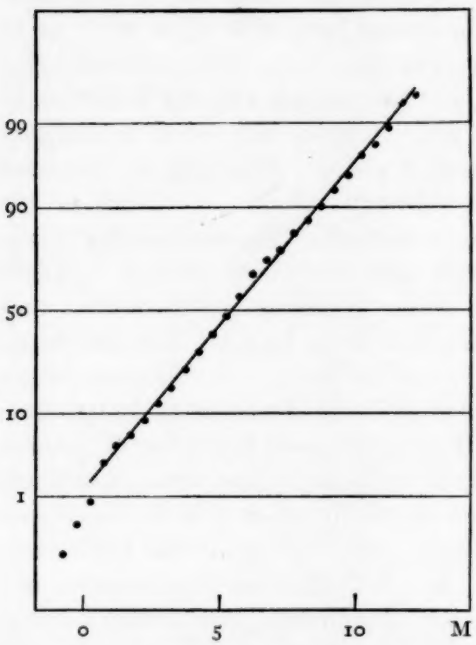


FIG. 4.—Frequency-curve of absolute magnitudes of stars nearer than 25.3 parsecs. Ordinates represent percentages of stars for which M is numerically less than the values indicated by the corresponding abscissae.

The distribution-curve of absolute magnitudes is shown in Figure 4, plotted on arithmetic-probability paper. Except for the very brightest stars, which form less than 2 per cent of the total, the distribution follows a normal error-curve as nearly as we may desire. Its constants, in the usual notation, are $h=0.29$, $M_0=5.5$. In Figure 5 the present results are plotted as percentages of Kapteyn's total density, and are compared with Kapteyn's luminosity law (full line). As is obvious, the present percentages fall much be-

low the theoretical values for the fainter stars. On the other hand, we might expect agreement for the very bright stars, where our list should be practically complete. But we notice a surprising excess of observed absolutely bright stars. This excess for the stars absolutely brighter than 3.0 is illustrated a little more fully in Figure 6, and is tabulated in Table II. Apart from being systematic, the differences involve such a large number of stars that it appears improbable that the discordances are due to chance. A repeated analysis of the parallaxes of the stars concerned indicates that possibly 5 but probably not more than 10 stars out of these 108 are really at distances greater than 25 parsecs, a number wholly inadequate to reduce the discordance to a chance effect. Agreement may be obtained by changing the constants of the luminosity and density laws, or by changing the functional form of the luminosity law. Corroborative evidence for the latter suggestion derived from the stars brighter than the eighth apparent magnitude will be presented later.

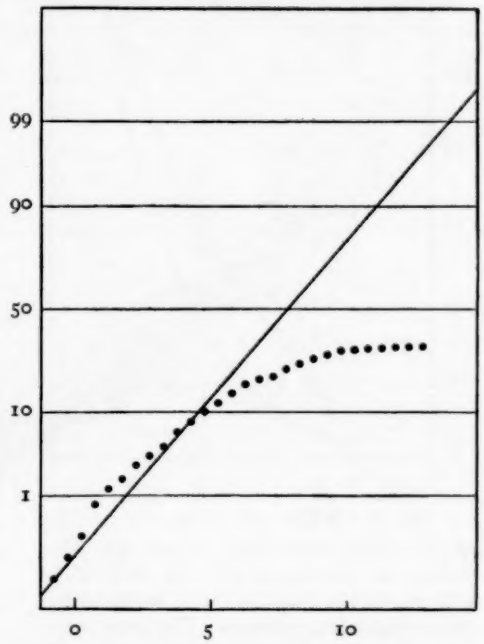


FIG. 5.—Frequency-curve of absolute magnitudes of stars nearer than 25.3 parsecs. Ordinates represent numbers of stars, taken as percentages of the theoretical total for which M is numerically less than the values indicated by the corresponding abscissae.

The relation between M and H for these stars is shown in Figure 7. Since many stars were included on the strength of a spectral and proper-motion parallax, the spreading of the dots is artificially diminished. The amount of diminution, however, is not very great,

as in these parallaxes the larger weight has been assigned to the value derived from the assumption of constant absolute magnitude for given spectral class. The reduction-curves used for the proper-motion parallaxes were different for each spectral subdivision, and the influence of these parallaxes on the slope of the M and H relation

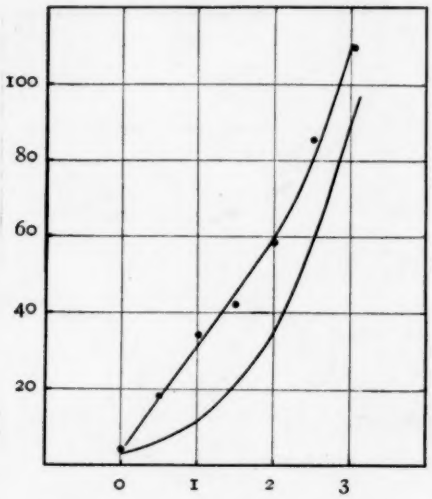


FIG. 6.—Bright end of the luminosity-curve of stars nearer than 25.3 parsecs. Ordinates are numbers of stars for which M is numerically less than the values indicated by the corresponding abscissae. The lower curve represents the theoretical Kapteyn curve.

TABLE II

COMPARISON OF THE NUMBERS OF STARS NEARER THAN 25 PARSECS, AND BRIGHTER THAN A GIVEN ABSOLUTE MAGNITUDE, AS COMPUTED FROM KAPTEYN'S LAWS AND AS OBSERVED

M	Kapteyn	Observed
0.0.....	3	5
0.5.....	6	18
1.0.....	11	33
1.5.....	21	42
2.0.....	35	56
2.5.....	59	84
3.0.....	93	108

must therefore be slight. Leaving aside the very bright stars which are statistically unimportant in the derivation of the luminosity law, we note again that the relation between M and H may with some justification be assumed to be linear when applied to all stars in space.

The constants of the correlation table are

$$r = 0.93 \pm 0.005$$
$$\sigma_M = \pm 2.4$$
$$\sigma_H = \pm 2.8$$

and the line of symmetry $M = -3.0 + 0.83\ H$.

The relation between spectral class and absolute magnitude is shown in Figure 8. The use of the spectral parallax is now largely responsible for the small dispersion in absolute magnitude along the dwarf branch. Three points may be noted:

1. The sudden drop in absolute magnitude between Ao and A2–3, which, however, may be due to chance.

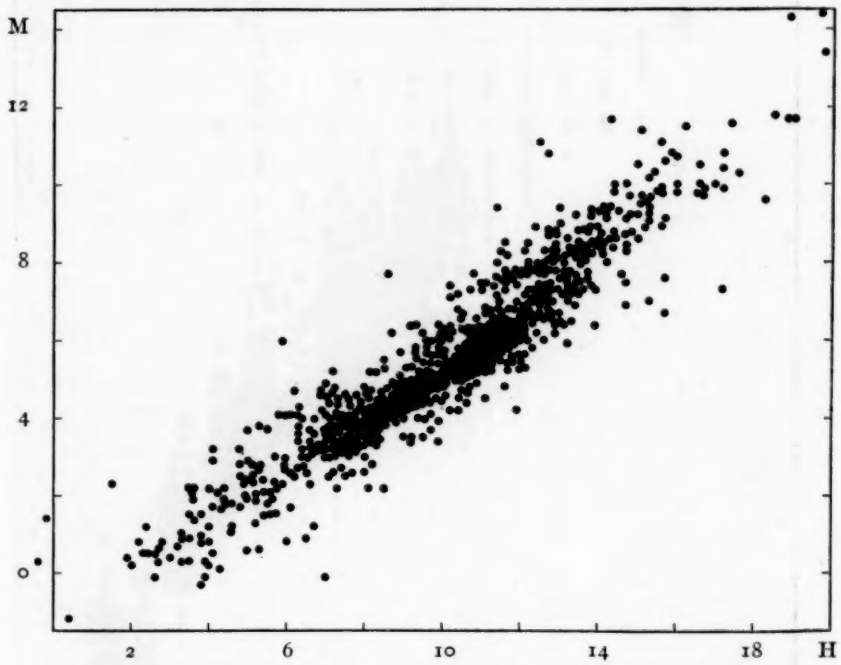


FIG. 7.—Relation between absolute magnitude, $M = m + 5 + 5 \log \pi$, and reduced proper motion, $H = m + 5 + 5 \log \mu$, for stars nearer than 25.3 parsecs.

2. The scarcity of giants of spectral classes intermediate between Ao and G8. Only two such stars are shown (α Ophiuchi and α Aurigae, both components) and an equal number of early B stars.

3. The absence of a "gap" between the K giants and dwarfs, as a result of which the yellow giants even appear to have "grown out" from the dwarfs. The numbers of stars involved being small, the appearance of the intermediary stars may be due to chance. Or, it may be due to erroneous values of the parallaxes used here. The fact that, in spite of the selective influences operating in such a way

as to favor the appearance of giants and dwarfs, our list contains even a few intermediary stars, lends support to the view that these

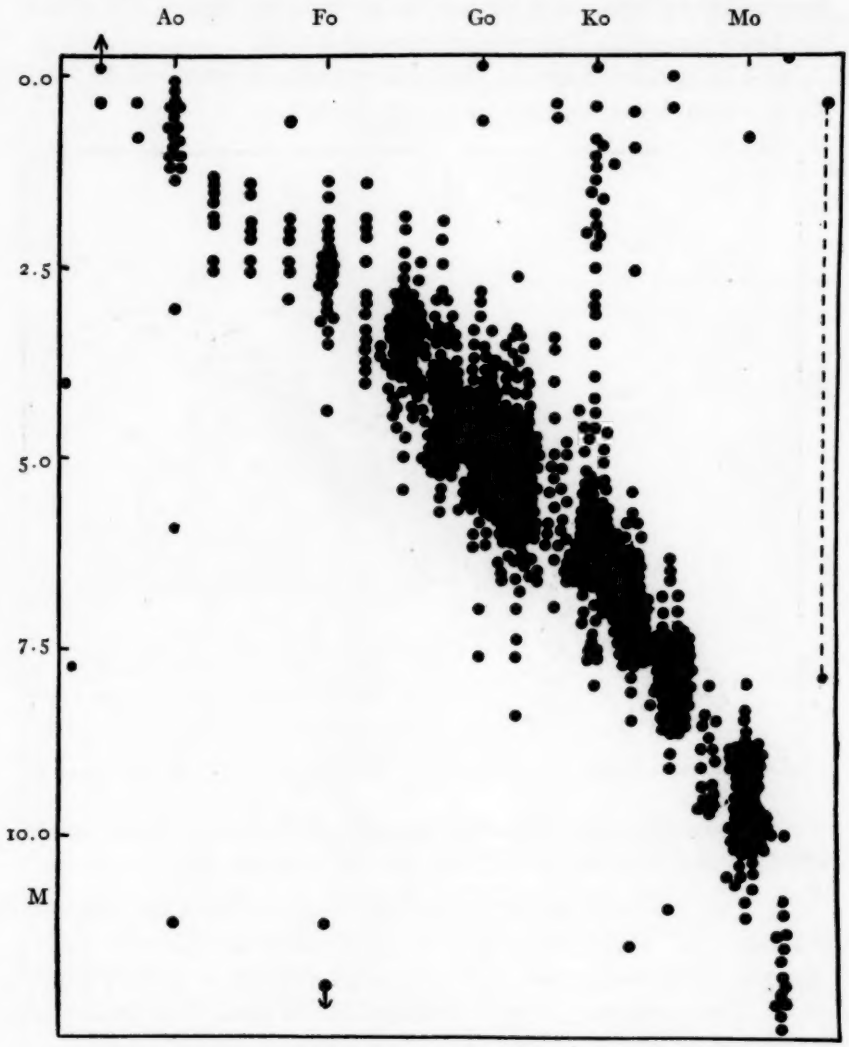


FIG. 8.—Relation between spectral class and absolute magnitude for stars nearer than 25.3 parsecs.

stars may be rather numerous in space. For purposes of future reference all K stars nearer than 25 parsecs, and brighter than 5.0

absolute, are collected in Table III. The last column contains the estimated probable errors of the absolute magnitude, which errors, it must be admitted, are personal estimates in many instances.

3. ABSOLUTE MAGNITUDES OF THE BRIGHT STARS

In order to examine more closely the brighter end of the luminosity-curve, we have made a comparison between the computed and observed numbers of stars brighter than a certain apparent

TABLE III

Star	R.A.	Dec.	Mag.	Spec.	π	M	ρM
α Boötis	14^h2	$+20°$	0.2	Ko	0".083	−0.2	±0.3
α Ursae Majoris	11.0	+62	2.0	Ko	.046	+ .3	.2
β Ceti	0.6	−19	2.2	Ko	.042	.3	.3
α Arietis	2.0	+23	2.2	K2	.045	.5	.4
ϵ Cygni	20.7	+34	2.6	Ko	.044	.8	.2
α Serpentis	15.7	+ 7	2.8	K2	.040	.8	.5
ϵ Scorpii	16.7	−34	2.4	Ko	.050	0.9	.5
γ Sagittarii	18.0	−30	3.1	Ko	.040	1.1	.5
θ Centauri	14.0	−36	2.3	Ko	.060	1.2	.3
ψ Ursae Majoris	11.0	+45	3.2	Ko	.042	1.3	.3
β Geminorum	7.8	+28	1.2	Ko	.105	1.3	.1
α Phoenicis	0.4	−43	2.4	Ko	.060	1.3	.4
τ Sagittarii	19.0	−28	3.4	Ko	.042	1.5	.5
λ Sagittarii	18.4	−25	2.9	Ko	.060	1.8	.5
η Herculis	16.7	+39	3.6	Ko	.045	1.9	.3
λ Andromedae	23.5	+45	4.0	Ko	.043	2.2	.2
γ Cephei	23.6	+77	3.4	Ko	.060	2.3	.2
ϵ Andromedae	0.6	+29	4.5	G5	.042	2.6	.4
η Serpentis	18.3	− 3	3.4	Ko	.068	2.6	.4
η Cephei	20.7	+61	3.6	Ko	.070	2.8	.2
28 Cassiopeiae	0.9	+59	4.8	Ko	.040	2.8	.9
δ Leporis	5.8	−21	3.9	Ko	.069	3.1	.5
β Aquilae	19.8	+ 6	3.9	Ko	.073	3.2	.3
37 Librae	15.5	−10	4.8	Ko	.049	3.3	.5
δ Eridani	3.6	−10	3.7	Ko	.105	3.8	.5
74 Draconis	20.9	+81	6.1	Ko	.043	4.3	.5
51 Pegasi	22.9	+20	5.6	Ko	0.065	+4.7	±0.5

magnitude whose absolute magnitudes lie between given limits. The computed number may be derived from Kapteyn's laws in the following way:

Again let $\Phi(M)dM$ be the number of stars with absolute magnitudes between M and $M+dM$, and let $\Delta(r, \beta)drd\beta$ be the number of stars per unit volume at a distance r from the sun, and in galactic latitude β, then the number of stars in the sky, brighter than appar-

ent magnitude m_c, and with absolute magnitudes between M and $M+dM$, equals

$$dN=\Phi(M)dM\int_0^{\pi/2}d\beta\int_0^R 4\pi r^2\cos\beta\cdot\Delta(r,\beta)dr$$

$$\log R=0.2(m_c-M+5).$$

Introducing ρ, the polar semi-diameter of the equidensity ellipsoids, and k, the ratio of its axes, we have

$$\rho^2=(\cos^2\beta+k^2\sin^2\beta)r^2/k^2\ .$$

Calling D the number of stars per cubic parsec near the sun, and furthermore, writing $\Delta(r,\beta)=D\cdot\Delta(\rho)$, we can express the integrand as an explicit function of r and β. Kapteyn's analytical function for $\Delta(\rho)$ leads to difficulties in integration; we can, however, represent his observed data fairly well by a formula of the form

$$\Delta(\rho)=1/(a^2\rho^2+b^2)\ .$$

We then have

$$dN=4\pi D\cdot\Phi(M)dM\cdot\int_0^{\pi/2}\int_0^R\frac{r^2\cos^2\beta}{(\cos^2\beta+k^2\sin^2\beta)a^2r^2/k^2+b^2}d\beta dr\ .$$

Substituting

$$\frac{a}{b}R=u\qquad 1-\frac{1}{k^2}=\lambda^2$$

$$\frac{1}{\lambda}\sqrt{1+\frac{a^2}{b^2k^2}}R^2=\sqrt{\frac{k^2+u^2}{k^2-1}}=q$$

$$\frac{4\pi k^2b}{a^3}D\cdot\Phi(M)dM=dC\ ,$$

we have

$$dN=dC(q\ \arctan\ u/q-\arctan\ u)$$

Tabulating dN for different values of M, we subsequently determine $\int_{M_1}^{M_2}dN=$ total number of stars in the sky brighter than the apparent magnitude m_c and between determined limits of M. Choosing $m_c=8.25$, down to which limit the *Henry Draper Catalogue* may be considered complete, and taking for M limiting values of $-4,-1$, and $-1,+2$, the theoretical number of stars appears to be 8800 in the former case and 34,000 in the latter. The observed numbers as shown in Table IV, which gives in order for every spectral subdivision the total number of stars in the sky brighter

than 8.25, the percentage and number assigned to group I, and those assigned to group II.[1] As Kapteyn's laws were derived without a discrimination as to spectral subdivisions, the argument presented here is probably not entirely circular.

The observed totals in each group are 3500 and 40,000, respectively. Many objections can undoubtedly be raised against the

TABLE IV

TOTAL NUMBERS OF STARS OF EACH SPECTRAL SUBDIVISION BRIGHTER THAN APPARENT MAGNITUDE 8.25; PERCENTAGES AND NUMBERS WHOSE ABSOLUTE MAGNITUDES LIE BETWEEN −4 AND −1 AND BETWEEN −1 AND +2, RESPECTIVELY

Spectrum	Number	P_1	N_1	P_2	N_2
B0–5	2061	85	1700	15	300
B8	1604	10	160	90	1450
B9	2752	10	275	90	2450
A0	6316	.....	.....	100	6300
A2–3	5240	.....	.....	100	5200
A5	1352	1	15	90	1200
F0	3208	3	100	50	1600
F2	1966	4	80	25	500
F5	3504	5	175	10	300
F8	2488	3	75	5	100
G0	2884	3	80	20	600
G5	5248	4	200	40	2100
K0	11,504	2	250	90	10,300
K2	4008	4	160	95	3800
K5	2832	5	130	95	2700
Ma	1204	10	100	90	1100
Mb–c	453	10	50	90	400
Total	.....	.....	3550	.....	40,400

arbitrary assignment of quota. It seems improbable, however, that the total number in group I should exceed 4000 or that in group II fall below 38,000. The conclusion may therefore be ventured that Kapteyn's luminosity law gives too many "supergiants," and not enough ordinary giants and early F dwarfs.

4. CORRELATION OF GALACTIC CONCENTRATION WITH THE LUMINOSITY AND DENSITY LAWS

Another indication that all is not well with the bright end of the luminosity-curve is given by the rapid change in galactic concentration between B8 and A5. On the basis of Kapteyn's

[1] *Harvard Circular*, No. 226, 1921.

density and luminosity laws there should be a definite galactic concentration for every absolute magnitude. The galactic concentration Γ may be defined as the ratio between the number of stars per square degree near the galactic equator to that near the pole of the galaxy. Instead of this we shall use the number of stars per square degree in the galactic belt $\pm 15°$, and that in the polar caps $\pm 55°$–$\pm 90°$, in order to make the results directly comparable with the observations of Shapley.[1]

Following the same argument as on page 24, we find the number of stars $dN(\beta)$ between galactic latitudes o and β, whose apparent brightness exceeds m_c and whose absolute magnitudes lie between M and $M+dM$, to be

$$dN(\beta)=2\pi\Phi(M)dM\int_0^R\int_0^\beta r^2\cos^2\beta\cdot\Delta(r,\beta)drd\beta\ .$$

Writing again

$$aR/b=u,\quad \sqrt{\frac{k^2+u^2}{k^2-1}}=q,\ \sin\beta=p,\ \sqrt{\sin^2\beta+1/k^2\cdot\cos^2\beta}=s\ ,$$

we obtain $dN(\beta)=F(M)\cdot(q\ \arctan\ pu/q-p/s\cdot\arctan\ su)$. Γ is then given by

$$\Gamma=0.689\,\frac{dN(15°)}{dN(90°)-dN(55°)}=f(M)\ .$$

Putting $m_c=8.25$, and using Kapteyn's constants, the following values of Γ are obtained:

M	Γ
−1.0	4.6
0.0	3.6
+1.0	2.6
+2.0	1.6

From Shapley's data we derive the following values of Γ:

10 for Ao, 4 for A2, 2.3 for A3, 1.9 for A5, 1.7 for Fo, 1.3 for F2 .

For the F stars, whose absolute magnitudes lie near +2, the agreement is satisfactory. For the earlier classes we can only reach

[1] *Harvard Bulletin*, No. 796, 1924.

accordance if we assign to them incredibly high luminosities. The inference is therefore that our assumptions about the density and luminosity laws are not correct, and it appears in particular to be the latter law which differs from the analytical form used here. We can explain the results by abandoning the idea of a universal law, and by making h a function of the galactic latitude such as $h=h_0(1+c \sin \beta)$. The value of c would have to be such that for the Ao stars, e.g., whose absolute magnitudes are about 0.0, the resulting galactic concentration becomes three times as high as before.

HARVARD COLLEGE OBSERVATORY
July 1924

THE FINE STRUCTURE OF THE NEAR INFRA-RED ABSORPTION BANDS OF WATER-VAPOR

By W. W. SLEATOR and E. R. PHELPS

ABSTRACT

With the advantage of better gratings it has become possible to extend previous studies of the absorption of radiation by water-vapor. The methods also represent a refinement of those previously employed, particularly in calibrating the spectrometer. The results of this study considerably enlarge our knowledge of the water spectra. Most important are measurements on the bands near 3.11 μ and 6.26 μ.

The work recounted in the present paper is a further investigation of the absorption of near infra-red radiation by water-vapor. This study, begun by Langley,[1] has been continued by Paschen,[2] von Bahr,[3] Sleator,[4] Hettner,[5] and others. The papers by von Bahr brought to light the surprising complexity of the absorption doublet described by Paschen near the wave-length 6 μ. The bolographs and tables given by Sleator showed that von Bahr had by no means revealed the ultimate details of this absorption band. They proved also that the absorption found by Langley, near the wave-lengths 1.3 μ, 1.8 μ, and 2.6 μ, was extremely, perhaps equally, complex. In neither of the four regions studied did Sleator complete the analysis, for while a few of the lines shown on his maps appear to be incapable of resolution by higher dispersion, more of them are still plainly combination effects. That is, one of them is not the graphical representation of the absorption of radiation of a single frequency. The spectrum of water, in short, is not yet as completely known as is the spectrum of hydrogen chloride by the work of Imes[6] and Colby and Meyer.[7] Since Sleator's paper was published, some improvements in

[1] S. P. Langley, "Researches on Solar Heat," *Professional Papers of the Signal Service*, No. 15, United States War Department, 1884.

[2] F. Paschen, *Annalen der Physik*, **51**, 4, 1894; **52**, 210, 1894; **53**, 234, 1894.

[3] E. von Bahr, *Verhandlungen der deutschen Physikalischen Gesellschaft*, **15**, 731, 1913; *Philosophical Magazine*, **28**, 71, 1914. Other references concerning the history of the subject are given in the latter paper.

[4] W. W. Sleator, *Astrophysical Journal*, **48**, 125, 1918.

[5] G. Hettner, *Annalen der Physik*, **55**, 476, 1918.

[6] E. S. Imes, *Astrophysical Journal*, **50**, 251, 1919.

[7] W. F. Colby and C. F. Meyer, *ibid.*, **53**, 4, 1921.

apparatus and technique have made possible an increase in resolution in the near infra-red spectrum and a corresponding increase in our knowledge of the absorption spectrum of water-vapor.

The apparatus was the double spectrometer, with thermopile and galvanometer, employed by Sleator and by Imes. The improvement was due entirely to better gratings. These are echelettes ruled by Barker at the University of Michigan. They served so well in concentrating energy in particular spectra that the slits of the spectrometer could be made much narrower than before, and the spectral interval affecting the thermopile at a particular setting of the grating correspondingly decreased. Numerical values for this interval will be given in connection with the particular spectra described.

The present procedure in securing data is better than the method described by Sleator, for, in the first place, the spectrum line of mercury at wave-length 1.014 μ was used in the calibration of some of the gratings. For others the band spectrum of hydrogen chloride near the wave-length 3.46 μ, which has been mapped with precision by Colby and Meyer (*loc. cit.* on p. 28), served the same purpose. Eight or ten lines of the *HCl* spectrum were used for an average value of the spectrometer constant. Moreover the "zero reading" of the spectrometer, necessary in finding every value of the angle in the equation $\lambda = K \sin \theta$, was determined bolometrically in every case. This determination was made at the beginning or end of every observation period. It was customary also to check the calibration constant K by using the actual grating space G in the equation $\lambda = 2G \cos \phi/2 \sin \theta$. Certain chances of error in calibrating by visual means have thus been eliminated.

In securing one of the energy-curves for the region around 6 μ, readings were taken at $\frac{1}{2}$-minute intervals. For all the other curves, the standard interval was 1 minute, except where the complexity of the pattern demanded more minute examination. If one considers the spectrum as spread out on a surface in which the thermopile slit forms a hole, the step taken between readings produces a displacement of the spectrum about equal to the width of the slit. In general, six deflections of the galvanometer were recorded for each point plotted on an energy-curve. Figure 2 is a tracing of the larger part

of one of three independent curves, necessarily somewhat smoothed out in the process of reproduction, and, in its present form, unsuited to the actual determination of wave-lengths.

This paper contains the report of work on the absorption regions

TABLE I

WAVE-LENGTHS OF LINES IN THE ABSORPTION BAND AT 6.26 μ. STARRED VALUES WERE CHECKED, USING $\frac{1}{2}$-MINUTE STEPS. μ IS THE UNIT

First Order	Second Order	Von Bahr	First Order	Second Order	Von Bahr
4.875			6.46	6.37	6.36
4.98	5.03	5.02		6.43	6.42
5.08	5.095	5.09			
	5.165	5.13	6.52	6.495	6.50
5.20	5.21	5.19		6.57	6.58
5.32	*5.295	5.27	6.615	6.62	
	*5.36	5.33		6.675	6.64
		5.40			
5.445	*5.42	5.44	6.72	6.72	6.72
	*5.475	5.50			
	*5.53	5.54		6.80	6.78
	*5.575	5.56	6.89	*6.875	6.87
5.60	*5.62	5.61		*6.92	6.91
	*5.65			6.975	6.97
5.69	*5.69	5.70	7.135	7.06	7.05
				7.165	7.17
	*5.74	5.74			
5.78	*5.77		7.29	7.21	7.22
	5.835	5.80		7.28	7.28
	5.88	5.86	7.335	7.315	
5.905	5.945	5.92		*7.39	7.36
	5.99	5.96			
			7.475	*7.425	
6.09	6.065	6.04		7.47	7.49
	6.11	6.10			
			7.625	7.59	.....
6.18	6.17	6.16		7.805	.....
	6.22				
	6.28	6.28	8.025	7.94	.....

near 6.26 μ, 3.11 μ, 1.87 μ, and 1.38 μ, which will be taken up in order. It is convenient to speak of one of these regions as a band, but it does not seem certain that any one of them is a band in the sense of the word which is applied to the spectrum of hydrogen chloride.

Our first work in the important region at 6.26 μ was done with an echelette grating having 480 lines per inch, using the first-order

spectrum. In the second order more absorption lines appeared, and the energy-curve was found to be very much like the one given by von Bahr (*loc. cit.* on p. 28), which was obtained with a prism. The two are given in Figure 1, and the corresponding wave-lengths in Table I. The probable error in these wave-lengths is 0.01 μ or 0.02 μ.

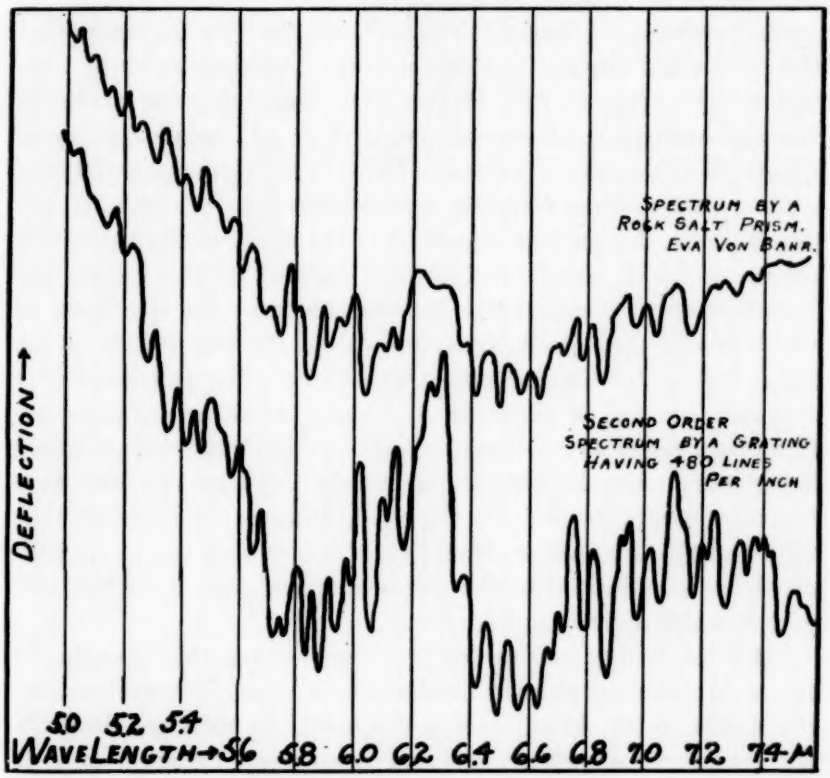


FIG. 1.—Water-vapor absorption near 6 μ

The interest of Figure 1 arises from the good correspondence of the maps made with instruments of different types. The wave-lengths in Table I bear out the similarity, though of course they do not represent our most accurate work. From the figure given by von Bahr it seems that an interval of about 300 A was included in the slit of the micrometer, or upon its receiving surface, and this is very nearly the value in the second-order grating spectrum under con-

sideration. Of course the significance of the number 300 A depends upon the purity of the spectrum, though the numerical value is determined by dispersion, focal length of mirror, and slit widths. The graph in Figure 1 is a free-hand copy of the map appearing in von Bahr's paper, reversed to correspond with our own, and it does injustice to the details of the original.

A comparison of the data of Table I will convince the reader that the grating is a valuable instrument in the investigation of the spectral distribution of energy. Its primary advantage, however, lies in the high dispersion at its command. This advantage appears in Figure 2 and the data of Table II. Here the numbers presented were secured with another echelette having 2880 lines per inch. It was possible with this grating to work over the whole of the absorption region, of which Sleator had covered only about two-thirds, and immediately to identify and find wave-lengths for 107 lines, of which Sleator had given values for 59. In the first review of the region the slit of the thermopile was $\frac{1}{2}$ mm wide, and included a spectrum interval of about 80 A. Finally, however, by using extreme care, the band was surveyed with the slits $\frac{1}{4}$ mm wide, readings being taken every half-minute of arc. In this way 126 lines were identified within the limits of the band. It appeared, however, that nearly all the new lines resulting from this re-examination were indicated by irregularities in the graph obtained with the wider slit, part of which appears in Figure 2.

It is interesting now to compare the wave-lengths appearing in this table with those given by Sleator (*loc. cit.* on p. 28) for the lines corresponding to the old numbers of column 2. In his paper the probable error in the wave-lengths of this region was put down as 20 A. It appears, however, that there is a systematic variation between his values and our later ones, the average difference for the 59 lines tabulated being 42 A, the older values being larger. This may be accounted for by the unsatisfactory method of calibration, for Sleator had to depend upon the value of the grating space and the measurement of an angle in the spectrometer, neither of which was reliable. However, if the average difference is subtracted from the older numbers, the variations seem to bear out Sleator's estimate of a probable error of 20 A, for 25 successive differences are 62, 59, 63,

TABLE II

WAVE-LENGTHS AND WAVE NUMBERS OF LINES IN THE ABSORPTION BAND AT 6.26 μ. THE ECHELETTE GRATING HAS 2880 LINES PER INCH
(A line whose relative intensity is 10 shows about 90 per cent absorption)

Line No.	Old No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)	Line No.	Old No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)
1	.....	1	209.12	4.7819	51	20	10	171.81	5.8204
2	.....	1	206.61	4.8401	52	.....	5	171.48	5.8316
3	.....	2	204.34	4.8940	53	.....	2	171.10	5.8447
4	.....	2	201.85	4.9542	54	20′	6	170.58	5.8622
5	.....	1	200.95	4.9763	55	21	10	170.08	5.8796
6	1	2	199.30	5.0176	56	21′	10	169.70	5.8928
7	.....	1	198.91	5.0275	57	.....	5	169.38	5.9039
8	.....	1	197.56	5.0617	58	.....	4	169.01	5.9168
9	2	2	196.80	5.0814	59	22	10	168.50	5.9349
10	3	1	196.20	5.0967	60	.....	4	168.14	5.9473
11	.....	1	195.57	5.1132	61	22′	5	167.59	5.9671
12	.....	1	195.06	5.1266	62	23	7	167.01	5.9878
13	.....	2	194.63	5.1377	63	24	4	166.37	6.0108
14	4	3	194.34	5.1456	64	25	10	165.43	6.0448
15	.....	1	193.54	5.1669	65	26	10	164.79	6.0683
16	5	1	193.02	5.1807	66	.....	8	163.81	6.1046
17	.....	1	192.66	5.1905	67	27	10	163.65	6.1107
18	6	3	192.39	5.1978	68	27′	3	162.90	6.1388
19	.....	3	191.90	5.2111	69	28	6	162.40	6.1576
20	7	2	190.98	5.2361	70	29	9	161.77	6.1816
21	7′	1	190.38	5.2527	71	.....	1	161.39	6.1960
22	8	1	189.59	5.2745	72	.....	2	161.03	6.2098
23	9	2	188.99	5.2914	73	30	1	160.78	6.2197
24	9′	1	188.55	5.3037	74	30′	2	160.36	6.2361
25	.....	1	187.77	5.3256	75	.....	1	160.17	6.2431
26	10	5	187.01	5.3474	76	31	2	159.56	6.2673
27	.....	1	186.23	5.3696	77	31′	1	159.02	6.2887
28	.....	1	185.88	5.3797	78	.....	1	158.72	6.3006
29	.....	1	185.40	5.3939	79	32	7	157.70	6.3413
30	11	5	184.52	5.4195	80	33	7	157.05	6.3675
31	11′	2	183.77	5.4416	81	33′	3	156.54	6.3881
32	12	4	183.02	5.4626	82	34	10	155.99	6.4105
33	.....	4	182.60	5.4766	83	.....	7	155.50	6.4309
34	.....	1	181.84	5.4995	84	35	4	155.11	6.4472
35	13	3	181.15	5.5204	85	.....	7	154.58	6.4690
36	13′	3	180.06	5.5537	86	36	10	154.07	6.4904
37	.....	1	179.65	5.5665	87	37	8	153.45	6.5169
38	14	4	179.26	5.5785	88	38	7	152.82	6.5438
39	.....	1	178.52	5.6016	89	.....	7	152.61	6.5525
40	15	2	178.08	5.6154	90	39	10	152.23	6.5692
41	16	6	177.33	5.6392	91	40	8	151.80	6.5878
42	.....	3	176.48	5.6666	92	.....	4	151.27	6.6108
43	17	3	176.26	5.6735	93	41	10	150.79	6.6319
44	17′	3	175.82	5.6875	94	.....	6	149.98	6.6675
45	18	5	175.00	5.7143	95	42	7	149.75	6.6778
46	.....	1	174.43	5.7330	96	43	8	149.10	6.7069
47	18′	2	174.10	5.7439	97	.....	5	148.77	6.7219
48	19	6	173.49	5.7641	98	44	2	148.23	6.7464
49	.....	1	173.10	5.7771	99	.....	4	147.78	6.7668
50	.....	1	172.41	5.8003	100	45	8	147.39	6.7849

TABLE II—*Continued*

Line No.	Old No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)	Line No.	Old No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)
101....	46	7	146.60	6.8215	114...	.....	6	139.55	7.1661
102....	.....	10	145.82	6.8574	115...	.....	6	138.80	7.2044
103....	.....	2	145.29	6.8830	116...	.....	3	138.33	7.2290
104....	.....	5	144.87	6.9027	117...	.....	1	137.83	7.2554
105....	.....	1	144.35	6.9274	118...	.....	4	137.47	7.2741
106....	.....	8	143.76	6.9560	119...	.....	3	136.93	7.3032
107....	.....	1	143.34	6.9764	120...	.....	4	136.35	7.3338
108....	.....	3	143.08	6.9892	121...	.....	1	135.55	7.3775
109....	.....	4	142.41	7.0220	122...	.....	5	134.00	7.4626
110....	.....	7	141.96	7.0443	123...	.....	2	133.71	7.4791
111....	.....	2	141.14	7.0850	124...	.....	2	131.94	7.5793
112....	.....	4	140.55	7.1149	125...	.....	2	131.31	7.6162
113....	.....	5	140.00	7.1427	126...	.....	1	130.11	7.6860

55, 46, 53, 46, 45, 54, 34, 46, 33, 25, 16, 38, 65, 45, 27, 31, 29, 36, 18, 34, 22, 31—all positive. Just one of the 59 wave-lengths is less than the later value, Number 16 in Table II; and one, Number 10, is very much (142 A) greater. In these cases the former wave-length does not seem to represent the same line as the later one. If we consider now that Sleator plotted only one energy-curve at 6 μ, while the present values depend on three, and that the later method of calibration is much more precise, it seems fair to estimate the probable error of the wave-lengths in Table II as ± 4 A.

It is plain that a refinement of analysis, which may cause two lines to appear where but one was listed before, is enough to make some of the wave-lengths appear quite wrong, while it may affect others not at all. In this connection it is to be noted that some of the lines of Figure 2 are sharp and definitely located, and appear to be the single effect of appropriate single causes. They may not prove to be separable into components, no matter what measure of resolution it becomes possible to apply. Such lines are numbers 38, 44, 61, 62, 68, 69, 70, 80, 81, and 101, for example. On the other hand, some of the so-called lines are probably complex, and their definite separation waits upon higher resolving power. Such are numbers 45, 51, 86, 93, and 102. It appears from this difference that some of the wave numbers in Table II represent radiation frequencies of definite significance in the process of light-absorption. They are valuable to as many significant figures as they may safely be carried. Other

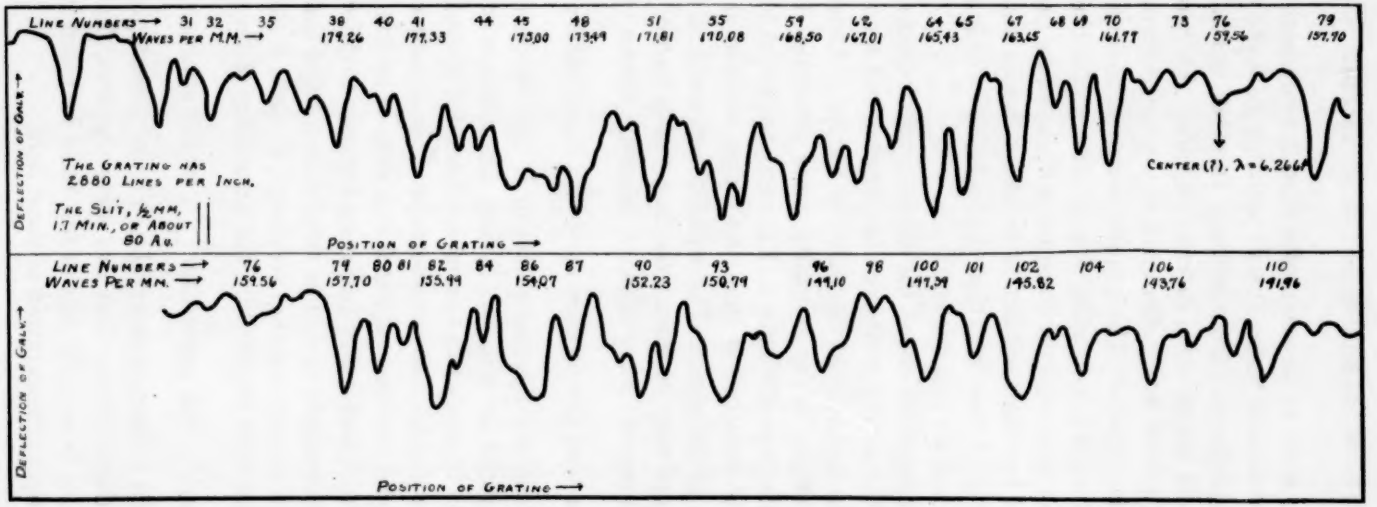
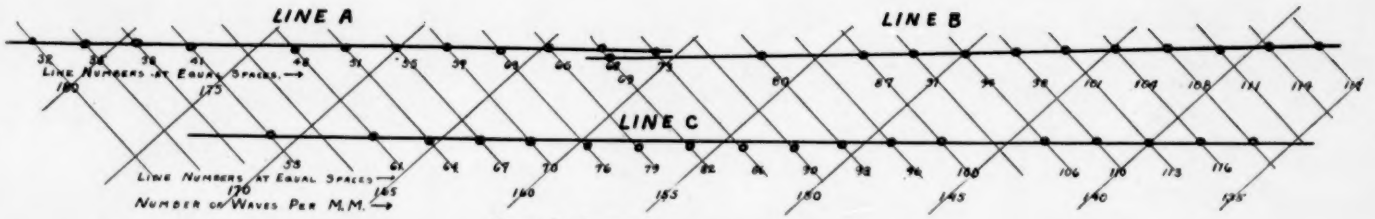


FIG. 2.—Part of the energy-curve, region of 6 μ

wave numbers seem to be composite effects, representing no single transition in molecular state. It will probably be futile to base theoretical calculations on these numbers. In other words, not all the data of Table II are of equal interest or value. An examination of the graph will permit a decision upon the value to theoretical work of any particular number. There must, of course, be some systematic error in all these wave-lengths, for no one would claim to have determined the spectrometer constant with absolute precision, but the value used has been subject to so many checks and redeterminations by different observers that the probable error ± 4 A seems to include variations on this account.

There is uncertainty about the significance of some of the measured lines in the actual process of absorption, but a certain degree of regularity is apparent in the curve of Figure 2. Lines 48, 51, 55, 59, 63, for example, occur at nearly equal intervals. This spacing suggests the diagram of Figure 3. The arrangement of Figure 3 with the co-ordinate axes rather than the graph inclined is a doubtful expedient for saving space. The equality apparent in the spacing along these lines may be fortuitous, but this is hardly likely. The three lines, however, have slightly different slopes, and, consequently, if one uses the formula $\Delta\nu = \frac{h}{4\pi^2 I}$ to calculate a moment of inertia, they indicate different values of I. The values appearing in the figure may not be real, for, if they are, a single corresponding change of state is related to absorption lines distributed among the whole set in no simple or obvious manner. It is, of course, impossible that there should be three principal moments of inertia represented by these numbers, and if they correspond simply to different stationary rotation states there should be more than three of them. Perhaps the two values closest together represent, in fact, a single one. At any rate, in the light supplied by our knowledge of the spectrum of hydrogen chloride, the expression $\frac{h}{4\pi^2 I}$ would appear to represent such a common frequency interval as Figure 2 most plainly shows. The fact mentioned above, that two wave numbers, though equally precise, may not be equally significant, must be taken into account in any theoretical consideration of these data.



Line	Av. Interval (Waves per mm)	Moment of Inertia	
A	1.838	$3.013 \cdot 10^{-40}$	gm. cm²
B	1.751	$3.157 \cdot 10^{-40}$	gm. cm²
C	1.828	$3.024 \cdot 10^{-40}$	gm. cm²

For *C*, $\Delta\nu = 3 \cdot 10^{11} \cdot 1828 = \frac{h}{4\pi^2 I}$

FIG. 3.—Linear relations between absorption lines, region near 6 μ

The work in the second region to be considered represents the first application to the band near the wave-length 3.11 μ of such high resolution as this apparatus permits, for Sleator's paper extended the band near 2.6 μ to the line which he numbered 87 at 2.86 μ, and he did not work any further toward longer waves. He supposed, in fact, that he had reached the end of the band. However, Hettner's map, in the paper already referred to on page 28, shows detailed absorption beyond the limit of Sleator's series. Also, in Hettner's paper on the band centers[1] the wave-length 3.15 μ is of particular importance as it represents the octave of the "fundamental" at 6.26 μ. The corresponding frequencies are Hettner's numbers ν_1 and $2\nu_1$.

This band was examined with a grating having 7200 lines per inch. The slits of the spectrometer were $\frac{1}{2}$ mm wide, which corresponded at the thermopile to about 32 A. The calibration depended upon the wave-lengths of 10 lines in the spectrum of hydrogen chloride as given by Colby and Meyer (*loc. cit.* on p. 28). Figure 4 and Table III present the data obtained. Lines numbered 84, 85, 86, and 87 at the beginning of the table are those so numbered by Sleator. Of these four lines, the former wave-lengths average 4 A (extremes 8 and -2) greater than present determinations, suggesting again a systematic error in the earlier calibration, decidedly smaller, however, than was made at 6 μ. Leaving out of consideration the fact that the application of higher resolution may show part of these lines to be complex, and the present wave numbers accordingly meaningless, it may be estimated that the probable error in the wave-lengths is ± 2 A.

It is interesting to compare Figure 3 with Hettner's map (*loc. cit.* on p. 28). In particular the center, if one may be guided by an appearance of symmetry, is near 3.11 μ rather than at 3.15 μ. In Figure 3 it may be noted also that the arrangement of lines is much less regular than in the other bands. In particular, one does not notice strong lines almost equally spaced, with weaker ones nearby on each side, which is an outstanding feature of the absorption near 6 μ. The band near 3 μ may accordingly depend upon a process or processes different from those which account for the others. It may, of course, represent a combination effect of common causes.

[1] G. Hettner, *Zeitschrift für Physik*, **1**, 345, 1920.

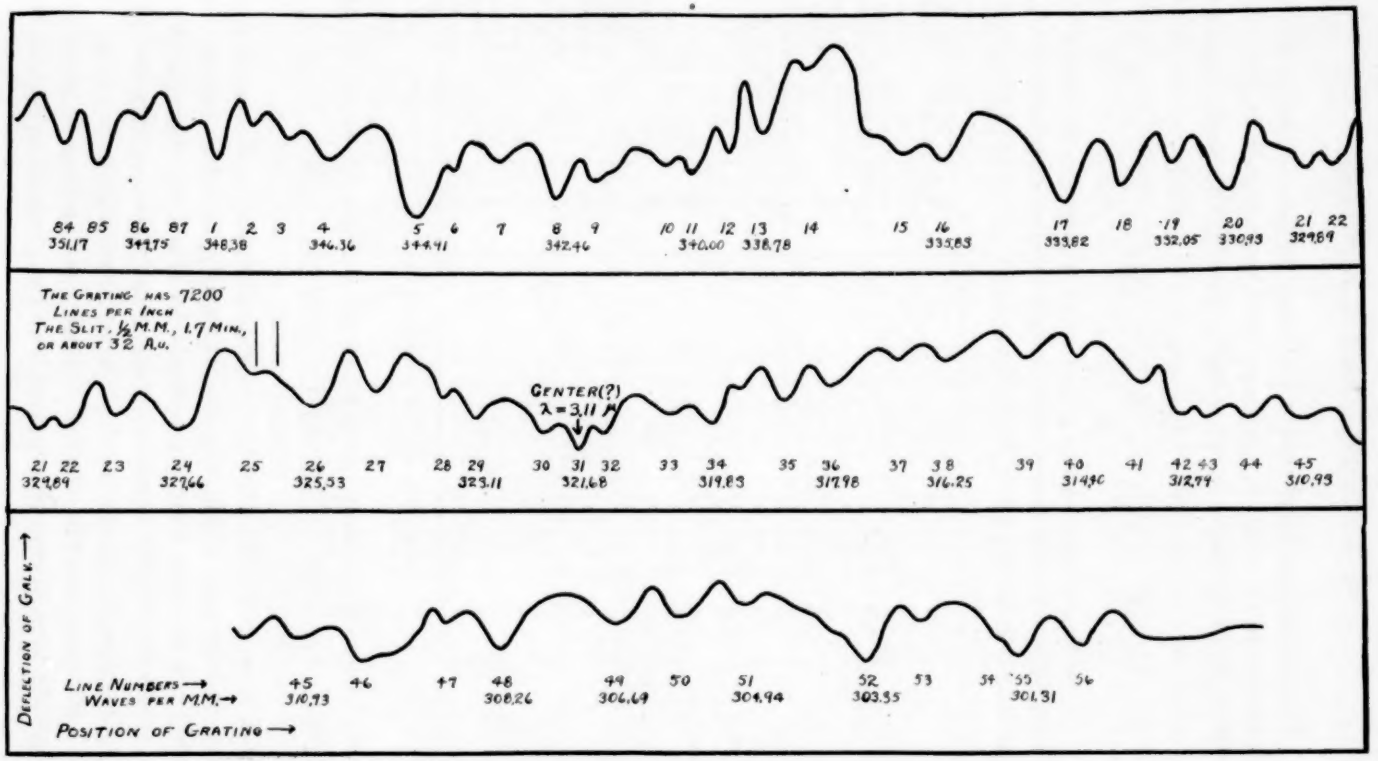


FIG. 4.—Energy-curve, region of 3.15 μ

The absorption band near 1.8 μ seems to have a center at line 28, whose wave-length is 1.8700 μ. This corresponds to Hettner's frequency $\nu_1+\nu_2$, where ν_1 and ν_2 refer, respectively, to the bands at

TABLE III

WAVE-LENGTHS AND WAVE NUMBERS OF ABSORPTION LINES IN THE BAND NEAR 3.11 μ

(A line whose relative intensity is 10 shows about 20 per cent absorption)

Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)	Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)
84	9	351.17	2.8477	27	3	324.72	3.0796
85	10	350.46	2.8534	28	5	323.63	3.0900
86	3	349.75	2.8592	29	6	323.11	3.0949
87	4	349.02	2.8652	30	6	322.16	3.1041
1	8	348.38	2.8704	31	6	321.68	3.1087
2	2	347.71	2.8760	32	3	321.11	3.1142
3	3	347.03	2.8816	33	1	320.26	3.1225
4	5	346.36	2.8872	34	3	319.83	3.1267
5	10	344.91	2.8993	35	5	318.66	3.1383
6	3	343.95	2.9075	36	5	317.98	3.1449
7	3	343.29	2.9130	37	1	316.92	3.1554
8	8	342.46	2.9207	38	3	316.25	3.1621
9	2	341.62	2.9273	39	3	315.10	3.1739
10	3	340.39	2.9378	40	3	314.40	3.1806
11	5	340.00	2.9412	41	2	313.41	3.1907
12	5	339.53	2.9452	42	1	312.79	3.1971
13	6	338.78	2.9517	43	1	312.43	3.2007
14	2	337.93	2.9592	44	2	311.84	3.2068
15	3	336.56	2.9712	45	3	310.93	3.2162
16	3	335.83	2.9777	46	5	310.11	3.2247
17	4	333.82	2.9956	47	1	309.04	3.2359
18	5	332.93	3.0036	48	5	308.26	3.2440
19	3	332.05	3.0116	49	6	306.69	3.2607
20	3	330.93	3.0215	50	5	305.75	3.2706
21	5	329.89	3.0314	51	3	304.94	3.2794
22	4	329.39	3.0360	52	5	303.35	3.2966
23	1	328.55	3.0437	53	1	302.63	3.3044
24	5	327.66	3.0519	54	3	301.61	3.3155
25	2	326.54	3.0624	55	4	301.31	3.3188
26	2	325.53	3.0719	56	3	300.52	3.3276

6.26 μ and 2.66 μ. In this band Sleator located and measured 30 lines, and the map of Figure 5 and Table IV give values for 52. The values of λ given by Sleator are, except for two, larger than the present numbers, the differences being not greater than 11, nor less than -1, average difference 5.5 A. The greatest differences (-1, 11, etc.) appear where the relative intensity of the lines is least, and where the present map shows more lines than Sleator could identify,

so perhaps the numbers compared really represent different things. However, there seems to be a systematic error in the former values of 4 or 5 A, an amount by which they are too large. If this systematic difference be deducted from the earlier values, the extreme variation will be ± 5 A between the earlier and later numbers, and it seems that the wave-lengths of Table IV should be affected by the probable error ± 2 A, or even a little less.

The grating used for these numbers was the one employed by Sleator, having 15,000 lines per inch, without the characteristics of

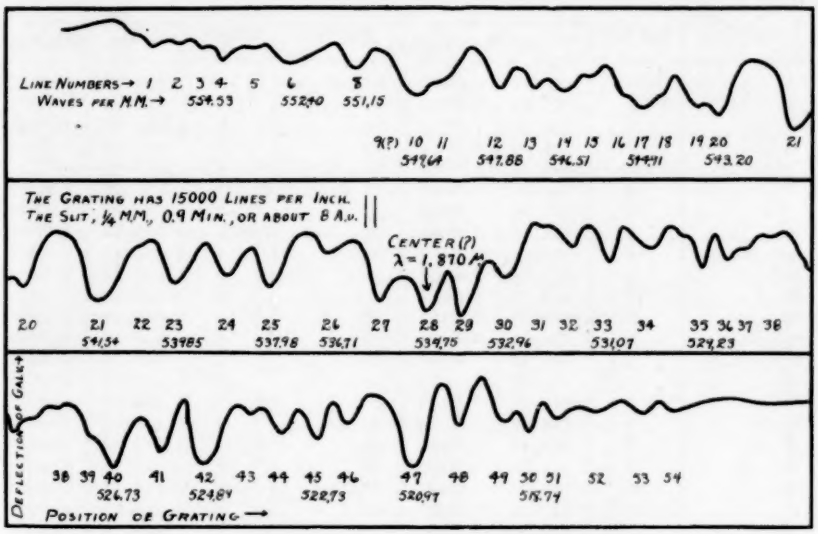


FIG. 5.—Energy-curve, region of 1.87 μ

the echelette. The slit of the thermopile included a range of about 8 A. If one compares the curve of Figure 5 with Figure 4 in Sleator's paper, it appears that the greater number of lines listed here is due largely to the confirmation of suggestions already noticeable in the earlier work but not at first considered reliable. The wave-lengths appearing here owe a somewhat greater accuracy to the better methods of measuring previously considered.

Such an improvement as has just been described has been made in the band at 1.3 μ also. Here 28 lines have been measured. The use of a grating having 20,000 lines per inch accounts for the larger

number—Sleator lists 14 lines. The band at 1.3 μ is, according to the present work, in about the same state of analysis as the band at 1.87 μ was left by Sleator—a comparison of his Figure 4 with Figure 6 shows this similarity. It is a natural question whether these absorption regions actually possess the complex structure to be ob-

TABLE IV

WAVE-LENGTHS AND WAVE NUMBERS OF ABSORPTION LINES IN THE BAND NEAR 1.87 μ

(A line whose relative intensity is 10 shows about 35 per cent absorption)

Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)	Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)
1	1	555.51	1.8002	27	10	535.77	1.8665
2	2	555.04	1.8017	28	6	534.75	1.8700
3	1	554.53	1.8033	29	8	533.85	1.8732
4	2	553.89	1.8054	30	6	532.96	1.8763
5	1	553.25	1.8075	31	1	532.57	1.8777
6	3	552.40	1.8103	32	3	531.76	1.8805
7	1	551.89	1.8119	33	4	531.07	1.8830
8	5	551.15	1.8143	34	3	530.18	1.8861
9	1	550.25	1.8173	35	5	529.23	1.8895
10	9	549.64	1.8194	36	3	528.77	1.8912
11	5	549.17	1.8214	37	2	528.36	1.8927
12	6	547.88	1.8255	38	1	527.79	1.8947
13	4	547.13	1.8277	39	2	527.18	1.8969
14	3	546.51	1.8298	40	8	526.73	1.8985
15	1	545.90	1.8319	41	7	525.75	1.9020
16	2	545.36	1.8336	42	9	524.89	1.9052
17	9	544.91	1.8352	43	1	524.07	1.9081
18	8	544.42	1.8368	44	3	523.44	1.9104
19	7	543.69	1.8393	45	7	522.73	1.9130
20	10	543.20	1.8409	46	3	522.11	1.9153
21	10	541.54	1.8466	47	9	520.97	1.9195
22	1	540.70	1.8495	48	4	520.05	1.9229
23	7	539.85	1.8524	49	1	519.18	1.9261
24	7	538.86	1.8558	50	2	518.74	1.9277
25	9	537.98	1.8588	51	1	518.16	1.9299
26	4	536.71	1.8632	52	1	517.44	1.9326

served at 6 μ, and what measure of similarity may ultimately be found in the details of the several bands. From the figures, the center at 6.26 μ is seen to represent small absorption, while in the other bands a point of maximum absorption is evidently the center. However, some of these regions may, and others may not, contain lines of a zero branch. Also, nothing more than a doubtful appearance of symmetry has determined our choice of centers, so they may or may not represent points of special, or in different regions of the same,

physical significance. It may be noted that, compared to the whole extent of the band, the individual steps by which the analysis was made—the value of $\Delta\nu$ corresponding to one of the equal displace-

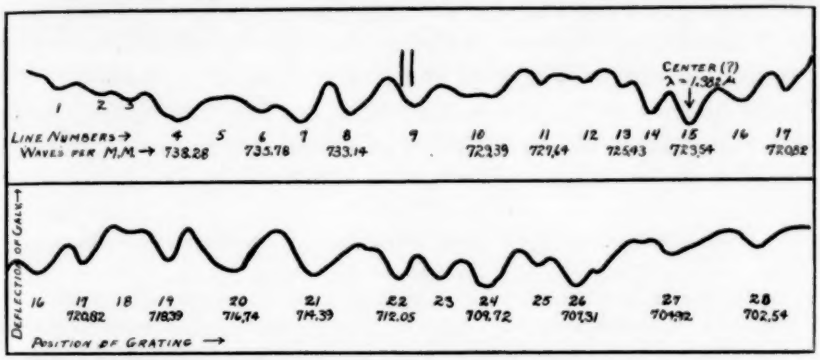


FIG. 6.—Energy-curve, region of 1.38 μ. The grating has 20,000 lines per inch. The slit, $\frac{1}{4}$ mm, 0.8 min., or about 7 A.

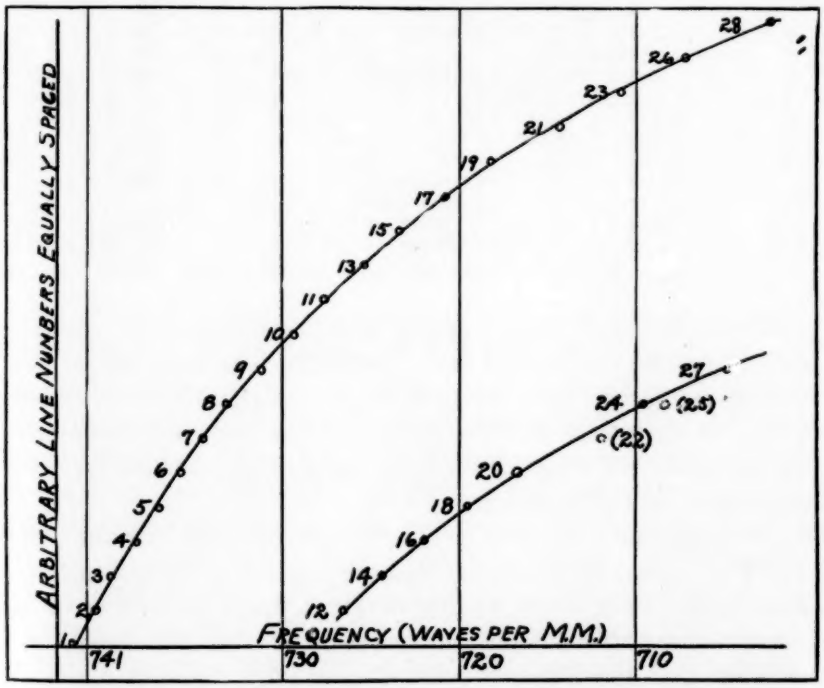


FIG. 7.—Regularities in the absorption band near 1.38 μ

ments of the grating—is larger at 1.35 μ than at 6.26 μ. There is, accordingly, a more refined analysis of the latter band. The energy-curve itself appears in Figure 6 and the corresponding data in Table V. In Figure 7 appears the same sort of graph as in Figure 3. The wave numbers are plotted on one axis, and the arbitrary line numbers are equally spaced along the other. Here, however, the resulting lines are by no means straight, but have a form suggesting the distribution of absorption lines in the spectrum of hydrogen

TABLE V

WAVE-LENGTHS AND WAVE NUMBERS OF ABSORPTION LINES IN THE BAND NEAR 1.38 μ

(A line whose relative intensity is 10 shows about 25 per cent absorption)

Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)	Line No.	Rel. Int.	Wave No. per mm	Wave-Length (μ)
1	2	741.84	1.3480	15	10	723.54	1.3821
2	1	740.52	1.3504	16	7	722.07	1.3849
3	1	739.64	1.3520	17	9	720.82	1.3873
4	7	738.28	1.3545	18	1	719.68	1.3895
5	1	736.92	1.3570	19	9	718.39	1.3920
6	5	735.78	1.3591	20	10	716.74	1.3952
7	9	734.48	1.3615	21	10	714.39	1.3998
8	10	733.14	1.3640	22	8	712.05	1.4044
9	9	731.26	1.3675	23	6	710.98	1.4065
10	6	729.39	1.3710	24	9	709.72	1.4090
11	2	727.64	1.3743	25	4	708.37	1.4117
12	1	726.59	1.3763	26	8	707.31	1.4138
13	3	725.43	1.3785	27	4	704.92	1.4186
14	7	724.53	1.3802	28	3	702.54	1.4234

chloride. Whether further analysis, if it becomes possible, will confirm or destroy this appearance of similarity it is impossible to say, but the regularity of the curves of Figure 7 can hardly be accidental. However, the intensities of the absorption lines vary in no systematic fashion along the series of Figure 7, which casts a doubt upon the importance of the alignment.

A comparison of these numbers with the table for the same region in Sleator's paper shows that the extreme differences in wave-length are 3 A and -1 A, average difference 1.4 A, the former values being higher. This difference varies so little, and is so small, considering that the numbers were obtained by different observers with different gratings, that one seems justified in setting the prob-

able error of the wave-lengths in Table V as 1 A. It does not seem likely that the same systematic error should remain in both calibrations.

It is possible now to compare the locations of the band centers as they are given by Hettner (*loc. cit.* on p. 38) with what appear to be the centers according to the data of this paper. The comparison appears in Table VI.

The first and third numbers in the second column do not agree as well as might be hoped with the observed values of the last col-

TABLE VI

LOCATION OF BAND CENTERS

Vibration Number	Wave-Lengths Computed from Data of This Paper for ν_1 and ν_2	Observed by Hettner and Others	Observed by Sleator and Phelps
ν_1	.....	6.26 μ	6.2673 μ
ν_2	.....	2.66	2.6720
$2\nu_1$	3.1337 μ	3.15	3.1087
$\nu_1+\nu_2$	1.8733	1.87	1.8700
$2\nu_2$	1.3360	1.37	1.3821
$3\nu_1$	2.0891	2.00, 2.05	.....
$2\nu_1+\nu_2$	1.4422	1.46	.....
$\nu_1+2\nu_2$	1.1013	1.13	.....
$3\nu_2$	0.8907	.....	.....
$4\nu_1$	1.5535	.....	.....
$3\nu_1+\nu_2$	1.1724	1.16	.....
$2\nu_1+2\nu_2$	0.9367	0.94	.....
$\nu_1+3\nu_2$	0.7898	0.77	.....
$4\nu_2$	0.6680	0.69	.....

umn. Also the spectral region between the bands at 1.38 μ, 1.87 μ, and 2.67 μ was carefully examined during this work, and no absorption comparable in intensity with that at 1.38 μ could be found with the vapor normally present in the air, which sufficed to show the bands themselves. The lines listed as near 2.00 μ and 2.05 μ appear in the sun's spectrum, and are attributed to water.

Some interest attaches to a comparison of wave-lengths and frequencies of absorption lines in the near infra-red with those of lines in the far infra-red. Data for such a comparison appear in Figure 8, in which the absorption lines are represented along a scale of wave numbers. For those measured in the present study—that is, in the near infra-red region—the number actually plotted represents

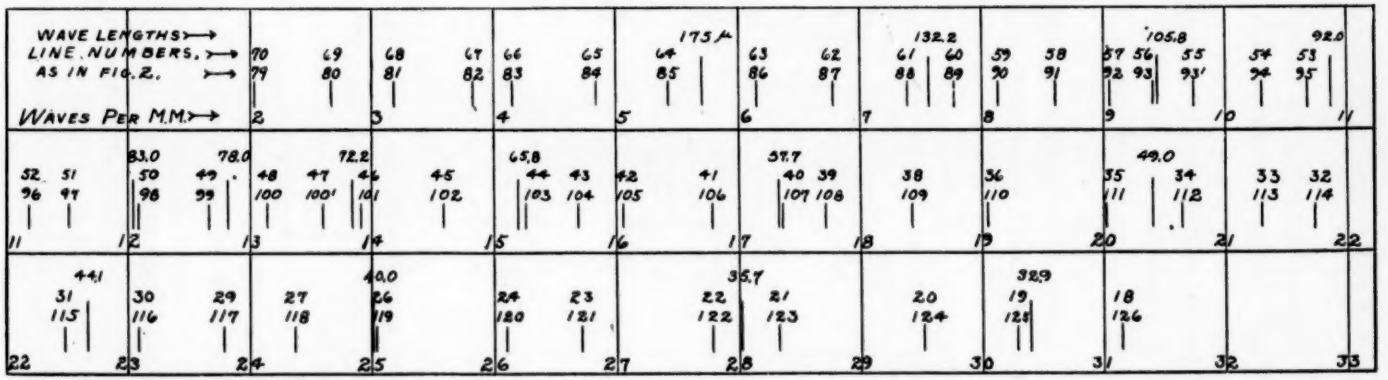


FIG. 8.—One-half the difference between the wave numbers of the lines of a pair, region of 6.26 μ, from Table II (short lines), and wave numbers of lines observed in the farther infra-red (long lines).

one-half the difference between the wave numbers of the two lines of a pair in the band near 6 μ. It must be noted that the correct location of the center of this band is by no means obvious, so that even what we call "a pair" may have no physical significance. We could only start near what appears to be the center and choose corresponding lines according to their spacing and appearance. In the case of lines nearest the center, the pairing seems from the curve to be real. The long lines of Figure 8 correspond to wave-lengths in the rotation spectrum as measured by Rubens and others.[1] These long lines occur in no regular order among the shorter ones representing the near infra-red. That is to say, the lines in the farther infra-red which make up the rotation spectrum have no obvious connection with what we have called the pairs in the band at 6.26 μ. In this band, however, there are so many lines that one could, with the rotation spectrum in view, so choose the pairs that the long lines of Figure 8 would fall either on or near the short ones, or halfway between successive short ones. Such an arrangement would be of interest only if one were sure that the corresponding relation between near and far infra-red ought to exist, or if one found that the numbers of each pair, chosen with such a relation in mind, were really similar and similarly placed in the graph of Figure 2. With the great number of lines available at 6.26 μ, one could find plausible indication of whatever relationship between near and far infra-red he wished to support.

It remains only to add that a great deal of work was done with absorption cells containing steam or carbon dioxide in order to check the actual responsibility of water for the absorption lines of the figures. In no case was a line enhanced by the tank of carbon dioxide, though it must have increased by a hundred fold the amount of that medium in the path of the radiation. The effect of the extra carbon dioxide was of particular importance near 1.38 μ, for a harmonic of one of the strong bands near 2.8 μ might have been expected in this neighborhood. No indication of such a harmonic could be noticed.

With a tank of steam in the path, all the lines were more intense,

[1] H. Rubens, *Sitzungsberichte der Preussischen Akadamie der Wissenschaften*, p. 3, 1921; p. 513, 1913. H. Rubens and G. Hettner, *ibid.*, p. 167, 1916.

on account, apparently, of the greater amount of water-vapor in the beam. Steam at various temperatures up to 350° C. was employed. The variation in temperature had no appreciable effect on the number, position, or relative intensity of the absorption lines. It might have been expected that a change of temperature would have shifted the maximum of absorption along the band, as is the case in the spectrum of hydrogen chloride, but no such effect could be observed.

UNIVERSITY OF MICHIGAN
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WAVE-LENGTHS AND PRESSURE-SHIFTS IN THE SPECTRUM OF MAGNESIUM

By MAX PETERSEN and JEROME B. GREEN

ABSTRACT

Existing pressure-shift data are considered by various writers to be combined with pole-shifts and perhaps density errors. Their relations to Stark effect are similarly qualified. It is proposed to try to eliminate these criticisms from new observations of magnesium.

Wave-lengths of some magnesium lines in an arc in air and an arc in vacuum are measured against iron standards in the second, third, and fifth orders of a 21-foot concave grating in a Rowland mounting. The sources and procedure are described in detail and the results of the measures of magnesium and iron lines are tabulated.

Pressure-shifts are deduced from the wave-lengths. The shifts are found to be smaller than existing older data, but not uniformly so. The shifts increase with series term according to some undetermined law. No violet shifts are determined. The shifts are in partial agreement with reported asymmetry of Stark effect, the two discrepancies involving small values.

The following observations of pressure-shift for magnesium were made with two considerations in mind: first, to attempt to meet certain criticisms of earlier pressure-shift data; and second, to find how the shift varies within a series and to see if it is in accord with the asymmetry of the Stark effect for those lines for which the latter is known.

Stark[1] has expressed the opinion that the pressure-shift is in substance an asymmetrical electric resolution of the line, occurring in the heterogeneous electric fields of the source rather than in the regular field of Stark effect researches. For this proposal Ritter[2] has assembled experimental support. From a comparison of some miscellaneous observations of pressure-shift and Stark effect he reached the conclusion that the substantial identity of the two phenomena was established. Ritter's conclusions are, however, subject to some qualification in view of recent criticisms of pressure-shift data.

The most important of these relates to the difference in wave-length between the light emitted from the center of the arc and that from the two poles, the so-called "pole effect." This is especially noticeable for short arcs. The propriety of recognizing practically

[1] Stark, *Annalen der Physik*, **43**, 1017, 1914; *Jahrbuch der Radioakt.*, **12**, 349, 1915.

[2] Ritter, *Annalen der Physik*, **59**, 170, 1919.

any of the present arc pressure-shift results as distinct from pole-shifts has come into serious question. Suspicion of this difficulty was, indeed, expressed by Royds,[1] and by St. John and Babcock;[2] and in the data of Whitney[3] and Miller[4] for the two effects in calcium one finds considerable justification for this suspicion. The observers have apparently shown that the anomalies of "pressure-shift" follow minutely those of "pole effect." For zinc the same conclusion is reached by Harris[5] from a comparison of his pole-shifts with the pressure-shifts of Swaim.[6]

The continued use of these two terms, "pressure-shift" and "pole effect," implies a distinction between the phenomena, even though it be admitted that both may have their origin in electrical forces. The description of the sources used by the observers mentioned leaves little doubt that the two phenomena have not been experimentally separated. In the observation of these pressure-shifts a short arc was used, for calcium a 4-millimeter arc of 5–6 amperes, and for zinc a 2.5-millimeter arc of 3.5 amperes. It is very doubtful that any part, even the center, of such an arc would be at all free from the disturbances which in parts of longer arcs cause pole-shifts. For the iron arc, standard wave-length requirements have dictated the use of an arc at least 12 millimeters long in order to avoid perceptible pole-shifts in the central millimeter zone. St. John and Babcock[7] estimate the average "pole-shift" at the center of a 6-millimeter iron arc to be 0.006 A. Consideration of these results obtained with short arcs leads to the conclusion that what was observed may have been largely, if not entirely, pole-shifts. Since the conclusions of Ritter's paper are based upon these same data and other earlier work of similar character, it seems necessary to qualify his conclusions in the same manner. The correspondence he finds

[1] *Kodaikanal Observatory Bulletin*, **3**, No. 43, 1914.

[2] *Astrophysical Journal*, **46**, 138, 1917; *Mt. Wilson Contr.*, No. 137.

[3] *Astrophysical Journal*, **44**, 65, 1916.

[4] *Ibid.*, **53**, 224, 1921.

[5] *Ibid.*, **59**, 261, 1924.

[6] *Ibid.*, **40**, 137, 1914. For calcium, Miller's pressure-shifts are pretty closely two-thirds of Whitney's pole-shifts. For zinc, Swaim's pressure-shifts and Harris' negative-to-center pole-shifts are nearly identical, his positive-to-center shifts being twice as great.

[7] *Loc. cit.*

between Stark effect and pressure-shifts may be due to combination of the latter with pole-shift. Other observers[1] have found such correspondence between pole-shifts and Stark effect for iron and nickel. The fruit of these considerations might cause serious doubt as to whether any true pressure-shift can be distinguished from pole effect were it not for observations of large pressure-shifts in furnace spectra.[2]

Feeling that these objections may vitiate any discussion of the nature of pressure-shift based on earlier observations, the authors have attempted to obtain sources from which the faults criticized were as far as possible removed. To minimize pole effect, a long arc in air was used instead of the customary short one. In order to reproduce conditions as nearly as possible, scant and similar amounts of vapor were used in the arcs in vacuum and in air. In order to avoid any prejudice or bias in measuring the shifts, they were not taken by direct comparison of vacuum and air spectra, but were deduced from separate wave-length determinations against iron standards, although this procedure multiplied the labor several times. At the same time, new determinations of the wave-lengths of certain *Mg* lines have been obtained which may prove of value.

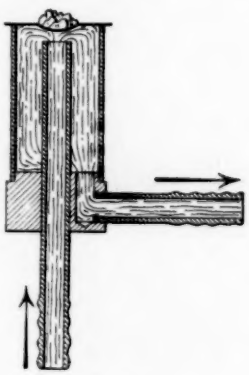

FIG. 1

The arc in air was devised after a suggestion by Hemsalech.[3] A bit of metallic magnesium was put on the thin copper top of the water cell of Figure 1. Almost as soon as the arc was struck, the metal burned, leaving the arc burning actually on the copper, but heating and supplying itself with a scant amount of *Mg* vapor. Although the cell was cooled by a generous stream of water, the arc eventually ($\frac{1}{4}$–2 hours) burned a small hole through the copper and a new top had to be quickly soldered on. The negative pole was a

[1] Takamine and Kokubu, *Mem. Coll. Sci. Kyoto*, **3**, 6, 173, 1918; *Astrophysical Journal*, **50**, 23, 1919; *Mt. Wilson Contr.*, No. 169.

[2] King, *Astrophysical Journal*, **34**, 37, 1911; **35**, 183, 1912.

[3] *Phil. Mag.*, **43**, 287, 1922.

half-inch carbon. The arc was burned customarily 30 millimeters long, less than two millimeters of the center zone being projected on the slit. It required longer exposures than did the vacuum arc for all lines observed, but considering the more favorable shape of the latter, it is judged that the magnesium-vapor densities were not greatly different in the two sources. This arc in air gave better lines than were obtainable from metal or oxide in carbon and was much more tractable. It burned steadily at about 5 amperes. The contamination of the magnesium spectrum by copper was for the authors' purposes quite unobjectionable.

The essential features of the vacuum arc were suggested by consideration of Fowler's[1] valuable paper on the spectra of magnesium. The construction of the arc is shown in Figure 2.[2]

The negative pole, *A*, to be viewed end on, was a rod of metallic magnesium clamped securely (for heat dissipation) in an articulated brass support so devised as to accommodate various lengths of specimens and to permit motions up and down and in the line of sight. The magnesium rod was surrounded by a tube of furnace magnesia or other similar material, constraining the arc to burn on the end of the metal. The operation of the arc spent the magnesium at a rate dependent on the current and the hydrogen pressure, making a pit of the magnesia tube, which could be used as deep as 2.5 centimeters or more. The support of this negative electrode passed through soapstone insulation *C*, the power lead coming to it through a heavy rubber bushing *D* and a flexible beaded conductor. The positive electrode was a machined button of magnesium, *B*, held very tightly in a water-cooled copper cup. The button of magnesium was made a snug fit and set into contact with the bottom of the cup. During operation of the arc the superior thermal expansion of the button with respect to the copper gave so tight a fit that the cup was permanently enlarged. The cup was soldered to the outer brass tube of the staff several centimeters below its tip. This shaft of tubing, through which was carried the water-cooling current, was grounded on the arc housing and extended through a stuffing-box permitting

[1] *Transactions of the Royal Society* (London), **214**, 225, 1914.

[2] The case in which the arc was enclosed is the same as used by Mendenhall and Forsythe, *Astrophysical Journal*, **37**, 380, 1913.

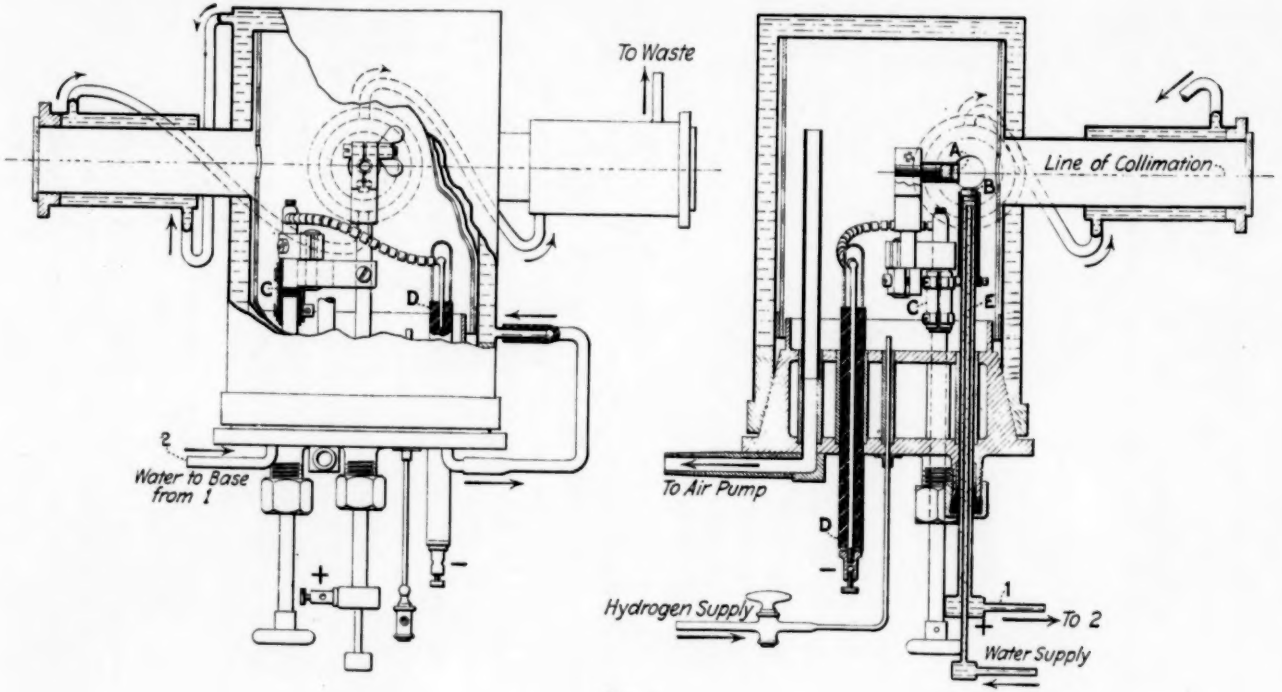
To Waste
Line of Collimation
A
B
C
D
E
2
Water to Base from 1
To Air Pump
Hydrogen Supply
1
To 2
Water Supply
+
−

FIG. 2

rotation and motion along the shaft. The arc was constrained to burn on the button by a jacket of furnace tubing extending above the copper. The operation of the arc either deposited *Mg* on this positive button or slowly spent it according to the current and pressure.

The arc operating in hydrogen at low pressure was so very steady in its behavior that considerable confidence is entertained that it is possible to repeat conditions of excitation with good accuracy. Extended observations of the spectral character of several parts of the arc were made by the authors in the study of another problem.[1] For the purposes of this paper, it is only important to note that within the length of the negative pit there was evidently a small potential drop, the voltage across the pit seemingly adding on to the electrode drop of a pitless arc and increasing the potential across the whole arc by about 20 volts per centimeter of pit depth, which was not much dependent on current value between 5 and 15 amperes. In the plates taken for wave-length of arc lines, the current was 5.0 amperes and the pressure from 0.3 to 1.0 millimeter, although a brighter discharge was sometimes obtained by using as much as 2 millimeters of hydrogen. The hydrogen was continuously pumped and pressure controlled by the inlet rate. The arc also operates very satisfactorily on negligible gas, using just enough current to give adequate magnesium vapor to carry the arc without its striking across to the support in preference to the deep pit.

The propriety of using a vacuum arc with the negative pole viewed end on for the authors' purposes is fortunately justified by the evidence of St. John and Babcock[2] that, for iron at least, the pole-shifts disappear in a vacuum arc. Furthermore, observations and measurements of the wave-lengths of the lines under various hydrogen pressures from 0.3 to 2 millimeters and with various depths of pit showed no difference in wave-length.

The iron arc used for wave-length standards was a Pfund arc with a carbon negative pole. On account of the chromatic aberration of the quartz lens used before the spectrograph slit, this arc was burned 30 millimeters long at 5 amperes current, standing in pre-

[1] *Astrophysical Journal*, **60**, 301, 1924.

[2] *Loc. cit.*

cisely the same location as did the magnesium arcs. The use of this long iron arc involves a peculiar and considerable weakening of some lines, and it sharpens up some other lines of doubtful behavior in shorter arcs. All three arcs were operated with suitable water-cooled resistances on a 320-volt circuit, this high voltage being found necessary for the long arcs used.

The light from the three sources was projected on the slit of the 21-foot concave grating by a simple quartz lens of about 25 centimeters focal length, producing an image on the slit about $2\frac{1}{2}$ times magnified. The slit was 4 millimeters long and open to a width of 0.013 millimeter, which is 4 times the Rayleigh $\lambda/4$ normal slit for this grating and λ 2800 A.

The grating has been remounted since the last report in an interior basement room of the new laboratory. The very substantial Rowland mounting was set up on piers independent of the building—piers standing on thick cushions of sawdust over cinders, which are on the hard clay soil. On their sides the piers are free from the earth, sawdust filling the gap after the forms were removed. The stability of the mounting is very satisfactory, only the most severe building-tremors being transmitted to it. The grating-room, which has gypsum walls, is surrounded save on one short side by thermostat-heated rooms. The temperature conditions are very satisfactory; at times when the lecture-room above is slightly warmer than the grating-room, the temperature often stays within three or four hundredths of a degree for a day at a time. It is not controlled, but is read on four thermometers about the mounting.

The adjustments of the Rowland mounting were carried out according to Kayser when the grating was installed, and after some further minute adjustments of the radius arm on its carriage, a constant radius focus was attained over practically the entire track. All the measurements reported are of spectrograms made with the grating belonging to this laboratory. Some observations were checked with a similar grating generously loaned by the Jefferson Laboratory, for which grateful acknowledgment is here made. The plates were mostly taken in the third order, second order being used for several of the fainter lines, and for the triplet λ 5167–83, of which the third order lay beyond the track. All exposures were made on

Cramer Crown plates ($2\frac{1}{2} \times 18$ inches) developed in ferrous oxalate, fixed in acid hypo, and dried on their long edges. Exposures varied from five minutes to two hours, and except for the shortest exposures, the iron spectrum was always photographed half before and half after the magnesium. Numerous reversed iron lines served to detect the least disturbance of the plate during an exposure. (Of all the plates inspected for measurement, seven were discarded.) No occulting shutter was used in front of the plate, the iron and magnesium spectra being completely superposed.

Standards of wave-length were chosen from the iron spectrum in the neighborhood of each magnesium group. Whenever available, secondary standards were used (the long arc left out a good number of these lines); supplementing these were included tertiary standards, only such being chosen as showed agreement to a single mill-angstrom among several published values. (During the course of the work the new Volume VII, 1, of Kayser and Konen was received, and it was found that all tertiary standards thus used agreed with their average values.) Other tertiary wave-lengths were measured merely for comparative reference, and of these the authors' wave-lengths are separately listed. Whenever standards of different order from the *Mg* line were used, correction to order was made, using the data of Meggers and Peters[1] on the dispersion of air.

The measurement of the plates was made on a Société Genevoise dividing machine having a 55-centimeter screw of 1 millimeter pitch. The head of the screw is graduated into divisions equivalent to 0.005 millimeter and carries a vernier reading to 0.0005 millimeter. A plate was measured six times across, setting once on each line (without retraction), then reversed left for right, and remeasured in the same manner. The placing of the several plates of a set on the machine was taken at random over a good part of the screw, thus somewhat to eliminate any errors of periodic nature, although none had been reported by others who have used the machine. Each set of plates for vacuum and air arcs was measured completely by one observer. In sets measured completely by both observers, the wave-lengths were in unexpected agreement. (Mean abs. dev., 0.0004 A; mean alg. dev., 0.00001 A). In setting on lines the effort was always

[1] *Sci. Paper, Bureau of Stand.*, No. 327; *Astrophysical Journal*, **50**, 56, 1919.

to locate the peak of intensity of the line, which for asymmetric lines is distinct from the "center of gravity" of the image.

TABLE I

Series	Line	λ (Vac.)	No. Plates	Order	λ (Air)	No. Plates	Order	Air-Vac.	Nacken (Air)
1p–1s	5167	.328	7	2	.339	8	2	+.011	.303
	5172	.694	7	2	.701	8	2	.007	.673
	5183	.616	7	2	.621	8	2	.005	.598
1p–2s	3329	.928	8	3	.937	8	3	.009	.934
	3332	.152	8	3	.164	8	3	.012	.163
	3336	.681	8	3	.692	8	3	.011	.688
1p–3s	2936	.744	4	3	.....	.....	.....	.....	.754
	2938	.478	6	3	.498	4	3	.020	.487
	2941	.999	6	3	2.020	4	3	.021	.016
1p–2d	3829	.357	6	3	.362	4	3	.005	.364
	3832	.303	6	3	.308	4	3	.005	.306
	3838	.291	6	3	.298	4	3	.007	.283
1p–3d	3091	.071	6	3	.081	6	3	.010	.093
	3092	.992	6	3	3.003	6	3	.011	.011
	3096	.897	6	3	.906	6	3	.009	.914
1p–4d	2846	.723	6	3	.....	.....	.....	.....	.....
	2848	.353	6	3	.386	4	3	.033	.38
	2851	.666	6	3	.674(?)	4	3	.008(?)	.....
pp′ group	2776	.691	4	3	.696	7	3	.005	.704
	2778	.272	4	3	.275	7	3	.003	.289
	2779	.834	4	3	.839	7	3	.005	.853
	2781	.418	4	3	.422	7	3	.004	.431
	2783	.973	4	3	.978	7	3	.005	.989
1P–4D	4702	.997	7	3	3.029	2 2	3 2	.032	.069
5D	4351	.912	4 2	2 3	.952	4	2	.040	.940
6D	4167	.216	4	2	.....	.....	.....	.....	.65
1S–1P	2852	.126	6	3	.138	4	3	.012	.128
1S–1p$_2$	4571	.096	6	2	.104	5	2	.008	.114
Spark lines 1σ–1π	2795	.532	2 3	3 5	.531	7 2	3 5	−.001	.545
	2802	.707	2 3	3 5	.708	7 2	3 5	+.001	.718
2δ–4φ	3104	.722	6	3	.....	.....	.....	.....	.....
	3104	.814	6	3	.....	.....	.....	.....	.....

The results of the measures of magnesium appear in Table I.

The iron lines measured, together with those used for standards (marked *n*), are listed in Table II.

Data on the Stark effect with which to compare the pressure-shifts are rather meager. Table III displays what is thus available. The Stark effect data are from Takamine[1] and from Stark and Kirschbaum.[2]

TABLE II

Line	P. & G.	K. & K Mean	Line	P. & G.	K. & K. Mean
3067	.251	.253	4132*n*	.063	.063
3075*n*	.726	.726	4143	.873	.873
3083	.746	.747			
3091*n*	.582	.582	4202*n*	.033	.033
3099	.973	.....	4271	.161	.....
			4271	.765	.765
3100	.309	.309	4282*n*	.408	.408
3100	.670	.671	4294	.131	.131
3125	.652	.....			
3134	.114	.....	4307	.907	.909
			4315	.090	.089
3305	.974	.978	4325	.767	.768
3306*n*	.358	.358	4375*n*	.934	.934
3370*n*	.788	.788	4383	.547	.549
3440	.609	.613	4415	.129	.128
3440*n*	.992	.991	4427*n*	.314	.314
3443	.879	.881			
3465	.865	.865	4547*n*	.853	.853
3475	.456	.456	4592	.658	.658
3476*n*	.707	.707			
3490*n*	.577	.577	4602*n*	.947	.947
3497	.843	.844	4667*n*	.461	.461
			4668	.147	.....
3513*n*	.821	.821			
3521	.264	.264	4707*n*	.288	.288
3526	.044	.....			
3526	.168	.....	4957	.311	.....
3820*n*	.430	.430	5006	.134	.....
3825	.885	.885	5012	.074	.073
3827	.827	.826			
3834	.226	.227			
3840	.441	.440			
3841*n*	.052	.052			

The most conspicuous feature that these data present is the positive shifts of the line 4571 and the 3838 triplet, which are reported to have violet-sided asymmetry in an electric field. The lines in question are not noticeably asymmetrical in the authors' arc in air, nor particularly diffuse. Especially 4571 was a beautifully

[1] *Loc. cit.* [2] *Loc. cit.*

sharp and symmetrical line in both of our sources. The probable errors of the wave-lengths of this line in vacuum and air arc were 0.0004 and 0.0007 A respectively. The diffuse triplet series starting at λ 3838 has been examined in an arc in air with large currents. A quartz spectrogram shows the later triplets of this series to have red asymmetry in several different exposures on different plates. What cause can be effective to produce a red shift of a line with a violet-sided Stark effect is a puzzle. It is possible that in strong fields a new component appears which is not present at all in weaker fields, as has been discovered by Koch to occur in helium. Such a component

TABLE III

Line	Stark Displacement A at 30,000 v./cm		Pressure-Shift in A/atm.
2795, 2802	very small to red		−.001, +.001
2852	very small to red		.012
3093	+0.50	+ .50	.011
3097	.75	.40	.009
3829	−0.08	−0.07	.005
3832	.11	.06	.005
3838	.12	.05	.007
4352	+0.60	+0.70	.040
4571	small to violet		+.008

would not affect in any sensible degree the wave-length of a line due to an arc in air.

A comparison of several of the authors' shifts with older data is possible. It should give an idea whether when "pressure" shifts are separated from "pole" shifts the data present any different aspect as compared to less careful observations. Such data are assembled in Table IV. It is seen that the earlier measured shifts are in general somewhat larger than the authors' values.

A consideration of the variation of pressure-shift through a series is not so fruitful as it is tempting. The functions that have been examined for relating shift to the series place of the line have yielded no simple or general relations. All experience indicates that any plot of shift in terms of a converging value of series position, either

in wave-length or wave-number, will be asymptotic to the convergence of the series. In the light of present atomic concepts, it should be more useful to think of line-shifts as a difference between the energy disturbances of the two orbits concerned with the emission. These orbital disturbances—shifts of spectral-terms—seem then more fundamental. Unfortunately, in no case have sufficient lines

TABLE IV

Line	Air-Vac. M. and P.*	Air-Vac. P. and G.
2776	.006 M.	+.005
2778	.006 M.	.003
2779	.006 M.	.005
2781	.006 M.	.004
2783	.006 M.	.005
2795	.006 M.	−.001
2802	.006 M.	+.001
2852	large	.012
5167	.010 M.	.011
5172	.010 M. .007 P.	.007
5183	.010 M. .010 P.	.005

* M = Mohler, *Astrophysical Journal*, **4**, 175, 1896; P = Perot, *Comptes Rendus*, **172**, 578, 1921.

been observed to permit an algebraic separation of the term-shifts. Having in mind the quadratic function of the quantic numbers that controls the hydrogen-term displacements in an electric field, the authors have tried several relations for shift in wave number in terms of the variable quantic number of the series.[1] The hope that such curves for the sharp and diffuse series might relate themselves so as to give the displacement of the p-term was not realized. The attempts have produced nothing definitive. Similar treatment of older data for calcium and zinc proved similarly unsatisfactory.

The authors' pressure-shifts for *Mg*, taken from sources from

[1] Catalán in *Nature*, **113**, 889, 1924; **114**, 192, 1924, has shown some regularities between curves for *Fe* and *Ti*, describing shifts of low-temperature multiplet lines in terms of the *sum* of the spectral terms (i.e., the average energy of the two orbits involved). In other words, he finds within a series quite the same translational similarity that would occur if the shifts were plotted against the variable term or the wave-number, which would seem a more logical procedure.

which a good part of the pole effect should have been removed, show some similarity to older data, but are in general smaller. The shifts increase with term number within a series; none are found to be sensibly negative. They are in partial accord with the reported Stark effect asymmetry, but in two cases definite red shifts are found for lines reported to have opposite electrical asymmetry. Magnesium behaves similarly to calcium in having especially small shifts of arc lines of doubly excited origin, of spark lines, and of its resonance line $1S-1p_2$.

The vacuum-arc wave-lengths are not discussed in detail pending their extension.

The authors wish here to express their grateful appreciation of the unfailing generosity with which they were provided the facilities for the conduct of this investigation.

UNIVERSITY OF WISCONSIN
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A THREE-DIMENSIONAL METHOD OF REPRESENTING QUANTUM TRANSITIONS IN BAND SPECTRA

By HARVEY B. LEMON and CHARLES M. BLACKBURN

ABSTRACT

A three-dimensional representation of transitions corresponding to radiations in band spectra constructed for the case of the first negative Deslandres system associated with carbon is described. The advantages of such a model are that in spite of the several hundred lines involved such important conceptions as initial and final states, the selection principle, and the relation of different subgroups of bands within the entire system are clearly shown.

In connection with atomic (line) spectra, energy-level diagrams have proved very helpful in the association of different stationary states of the atom, and those spectral lines corresponding to transitions between them. Such diagrams vary from comparative simplicity, in the case of hydrogen, through advancing degrees of complexity for the spectra of the elements of the third group where many levels may be involved.

The representation of energy-levels in any two-dimensional diagram for the much more complex molecular (band) spectra, consequently, although obviously possible, is well nigh useless since some seven or eight hundred levels will be required.

Any energy-level representation emphasizes particularly: (1) the spectral "terms" which are the items of chief interest in the classification of spectra, rather than the frequencies themselves which may always be represented as the difference between two terms; (2) the existence of "stationary states" which result from the quantization of the motions reponsible for the radiation, although they in general give no hint as to how such quantization may be accomplished.

In connection with the analysis of the first negative Deslandres system of bands associated with carbon, recently reported,[1] a model showing the quantum transitions responsible for the observed lines has been devised. While not an energy-level diagram

[1] *Proceedings of the National Academy of Sciences*, **11**, 28, 1925.

in the quantitative sense, the stationary states, the Bohr frequency condition, and even the selection principle are conveniently displayed. Also, the relationships of the individual members of different groups of bands are made clear for a system in which these relationships in the linear diagram are quite obscure.

According to the current theory using the notation of Kratzer, the energy of a rotating and vibrating di-pole molecule is given by

$$W_m^n = nh\nu_0(1 - xn + \ldots) - m^2 anh + \frac{h^2}{8\pi^2 J}(m^2 - \ldots);$$

n and m are quantum numbers associated with the vibrational and rotational states, ν_0 is the frequency of vibration of the poles of the molecule, x is a small constant depending on the anharmonic bond between the poles, a is a small constant concerned with the effect of the oscillation on the rotation, J is the so-called "moment of inertia" of the molecule, h is Planck's constant of action.

Visible bands have their origin in the superposition of three quantum transitions: (*a*) Electronic. Aside from higher-order terms this transition may be regarded as a mere change in the J from J' to J. (*b*) Vibrational. This transition involves a change in the n from n' to n, with no restriction on the size of the jump provided the vibration is anharmonic. (*c*) Rotational. This is represented by the transition in m from m' to m restricted, however, by the selection principle $m' - m = \pm 1$ or 0.

Thus, for the representation of a single-band system, i.e., for a single electronic jump, we specify the energy by (1) J', n', m', for the initial state; (2) J, n, m, for the final state, where the n's and m's are variables.

The essential idea in this three-dimensional model is therefore the selection of two planes—one for the initial state and one for the final state. In the former, values of n' are plotted as abscissae and m' as ordinates. In the latter, the quantities n and m are so used. These two planes are placed parallel to one another at an arbitrary distance apart.

A transition representing the radiation of any spectral line is then represented by stretching a thread from the proper n'- and m'-

point in the initial plane to the corresponding n- and m- point in the final plane.

The various lines of a single band are thus given by a series of threads lying in a *plane* intersecting the initial and final planes in vertical lines for which n' and n are constant, respectively. In the band system for which this model was constructed, the parabolas have no Q-branch, that is, $m'-m=\pm 1$ only. This appears in the model by the fact that in this vertical plane of threads there are none passing horizontally from any given m' directly across to the oppo-

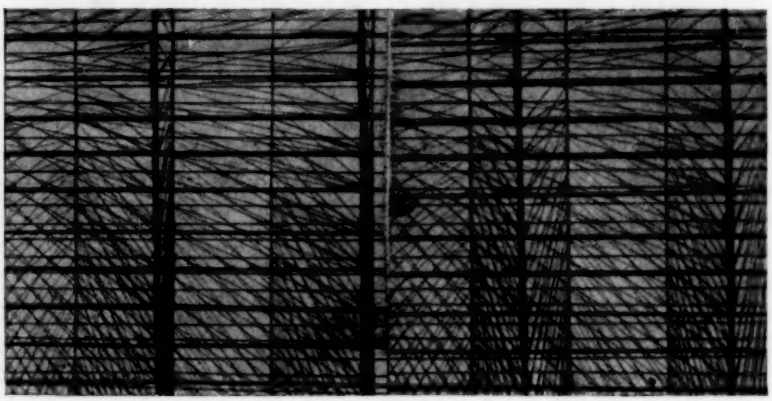

FIG. 1

site (same numerical value) of m, but from each point m' two threads only run across to those two points which are respectively greater and less by one numerically than the point of their origin. This crossing over of the threads is shown in the photograph (Fig. 1) printed stereoscopically for the benefit of those who have acquired the simple trick of viewing such prints directly from the printed page.

The various bands which are associated in a group of the entire system and which involve equal jumps in n from the initial to the final state ($\Delta n=n'-n=$constant) are represented by a series of parallel planes of threads in the model. These are shown in Figure 2, which is photographed at a greater distance and in a direction 90°

from Figure 1, i.e., parallel approximately to the planes of the threads.

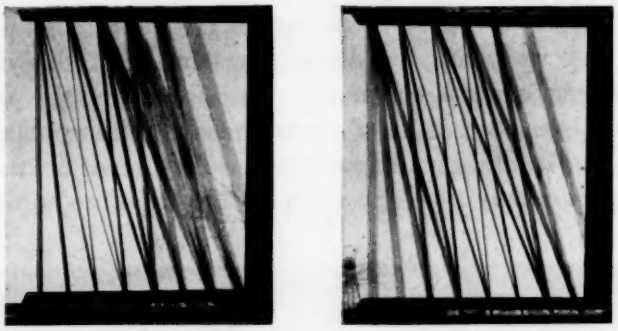

FIG. 2

Different groups of bands are represented by planes of threads crossing the model in different directions, and hence the interlacing of the planes of the entire system.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
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REVIEWS

History of the Royal Astronomical Society, 1820–1920. Edited by J. L. E. DREYER and H. H. TURNER, with chapters by them and by R. A. SAMPSON, the late E. H. GROVE-HILLS, H. F. NEWALL, and H. P. HOLLIS. London: Published by the Royal Astronomical Society and sold by Wheldon and Wesley, 1923. Pp. vii+258. 12 portraits. £1.

No man of science can fail to be stirred by the history of the great national societies, which prepared the way and were prime factors in the final establishment of the present era of research. The leaders of Greek thought, working for centuries at Alexandria and sharply contrasting with their Egyptian and Chaldean predecessors in the conception of their problems and in the observational and mathematical methods devised for their solution, arouse our interest in the first national academy, the fountainhead of true science. The Arabian centers of learning, from Bagdad to Cordoba, if less easily visualized, command our respect for those who carried on science and research when most of Europe was at its lowest intellectual ebb. The great days of the Accademia dei Lincei and the Accademia del Cimento, with the vigorous and prolific Galileo as their leading spirit, bring us out of medievalism into the modern world of progress. The Paris Academy of Sciences, at first composed exclusively of mathematicians and astronomers, brilliantly reflects the advance in Western Europe. In England the Royal Society of London, chartered in 1663, also devoted great attention to astronomy, and kept Isaac Newton in its presidential chair for twenty-seven years. Until 1788, when the Linnean Society was established, the Royal Society covered for England the entire field of knowledge. In his *Memoirs*, however, Augustus de Morgan quotes Sir John Herschel as follows:

> The end of the eighteenth and beginning of the nineteenth centuries were remarkable for the small amount of scientific movement going on in this country, especially in its more exact departments. . . . . Mathematics were at the last gasp, and Astronomy nearly so—I mean in those members of its frame which depend upon precise measurement and systematic calculation. The chilling torpor of routine had begun to spread itself over all those branches of science which wanted the excitement of experimental research.

Thus the founding of the Royal Astronomical Society in 1820 occurred at a critical time, not lacking in observational activity—though Sir William Herschel's epoch-making work was nearly done—but defective from the standpoint of precision of measurement and weak in mathematical theory.

The formation of an Astronomical Society, urged in print by Baily in 1819, was first suggested by Dr. William Pearson as early as 1812. He was one of the fourteen members present at the initial meeting, held in connection with a dinner at the Freemason's Tavern on January 12, 1820. In an address on the purposes of the Society prepared by Sir John Herschel, and edited (not wholly to his liking) by the secretary, Francis Baily, the following passage indicates one of the ambitious objects in view:

> One of the first great steps towards an accurate knowledge of the construction of the heavens is an acquaintance with the individual objects they present; in other words, the formation of a complete catalogue of stars and other bodies, upon a scale infinitely more extensive than any yet undertaken; and to be carried down to the minutest objects, *not* visible in any but the very best telescopes.

Another passage, stricken by Herschel from his MS and again from Baily's version of it, would have multiplied the difficulties of this colossal task:

> Yet it is possible that some bodies, of a nature altogether new, and whose discovery may tend in future to disclose the most important secrets in the system of the universe, may be concealed under the appearance of very minute single stars no way distinguishable from others of a less interesting character, but by the test of careful and often repeated observations.

In commenting on this omission, Professor Turner points out that Sir John Herschel's insistence was probably due to his appreciation, not adequately shared by his enthusiastic colleagues, of the impossibility of completing their announced plan. He cites the Astrographic Chart, initiated in 1887 and still far from accomplishment, as a similar illustration of too ambitious endeavor.

The moral is a useful one for later generations to ponder, but fortunately the new-fledged Society amply succeeded in accomplishing most of the other objects proposed in Herschel's address. These included the discussion and publication of observations, the education of observers, the determination of the latitude and longitude of various stations, the improvement of lunar tables, the cultivation of relations with foreign

astronomers, the diffusion of information, the computation of orbits, the proposal of prize problems, and the formation of a library. This last activity is one for which many hundreds of readers have reason to be grateful. Perhaps I may venture to voice the thanks of visiting astronomers, who have been given the freest and most courteous access to this rich storehouse, which contains such a varied and complete collection of books and journals relating to every branch of astronomy.

The monthly meetings of the Society, which also date from its origin, have proved to be an even richer legacy of its founders to the astronomical world. The complete papers published in the *Monthly Notices* and the *Memoirs*, and the reports of the discussions which appear so promptly in the *Observatory*, are read everywhere, and serve as the best possible means of maintaining contact with recent advances. The discussions are not of the perfunctory sort, and their preservation, revised as they are by the various speakers, adds greatly to the value of the papers themselves.

If space permitted, many pages might advantageously be devoted to excerpts from the interesting chapters on the succesive decades in the Society's history, prepared for this volume by the six joint authors. An early achievement was the publication of Baily's *Catalogue* of nearly three thousand fixed stars, which appeared as an appendix to the second volume of the *Memoirs*. Professor Turner, in his valuable account of *Our Founders*, describes Baily as "the backbone of the Society throughout its early years," and gives many interesting details from Baily's *Journal of a Tour in Unsettled Parts of North America in 1796 and 1797*, edited after his death by Augustus de Morgan. His experiences when his boat was frozen in near the bank of the Ohio River, at temperatures far below zero, sound like those of a polar explorer. Though tempted first to become an American citizen and later to continue a life of adventure in other lands, Baily returned to England and devoted himself to the interests of the Society until his death in 1844. Turner describes several other founders, and gives a contemporary record of a visit to the picturesque owner of Hartwell House, "the learned Dr. Lee," familiar to readers of Admiral Smyth's *Aedes Hartwellianae*.

The first volume of the *Memoirs* appeared in 1821. The *Monthly Notices*, which for some years consisted of brief notes in the *Philosophical Magazine*, were first published in separate numbers in 1827. These were reprints, but in 1834 the Council undertook the printing and publication of the *Monthly Notices* independently of the *Magazine*. The first gold medals of the Society were presented in 1824, one to Babbage

for his calculating machine and one to Encke for the determination of the elliptic orbit of the comet which bears his name. Two silver medals were also given for comet discoveries. The progress of astronomy in the United States is indicated by the fact that fifteen of the forty-six gold medals awarded by the Society during the last half-century have been given to American astronomers.

Dr. Dreyer's first contribution covers the decade 1830–1840. The long negotiations which resulted in the granting of a charter in 1831 brought into prominence the irascible Sir James South, whose peculiarities, illustrated when he smashed his telescope mounting to bits because "Struve has reaped the golden harvest among the double stars and there is little now for me to hope or expect," were recalled by Burnham in the Introduction to his *Yerkes Catalogue of Double Stars*. The reform of the *Nautical Almanac* then engaged the attention of the Society, under the leadership of Airy and Herschel. The rise in England of astronomy of precision during this period, after a long interval of stagnation, is chiefly associated with the name of Airy. His admirable work at the Cambridge Observatory, where he gave special attention to the positions of the planets, led to his appointment as Astronomer Royal in 1835. At Greenwich he effected great and lasting reforms, and continued on a larger scale the work begun at Cambridge. Another outstanding achievement of this period was Henderson's discovery of the parallax of α Centauri, which was not published until January, 1839—two months after Bessel's announcement of the parallax of 61 Cygni. Mention should also be made of Baily's investigations on the figure and density of the earth and on standards of length, the study of instrumental errors by Sheepshanks, Robinson, and others, the publication of Airy's *Cambridge Catalogue of 726 Stars*, and the important work in both hemispheres of Sir John Herschel.

As Dr. Dreyer points out, Sir John Herschel's examination of the northern and southern heavens, during a period of thirteen years, is an undertaking unique in the history of science. His discoveries at Slough alone comprise 3346 double stars and 2306 nebulae and clusters, and his observations during four years at the Cape were on a corresponding scale. One of his legacies to modern astronomy is a photograph on glass of his father's 40-foot telescope, taken in December, 1839, just prior to its demolition—a remarkable forerunner of the photographic methods which were soon to transform astronomy. Apart from Herschel, Baily, Dawes, Maclear, Smyth, and two or three others, there were comparatively few amateur astronomers at this period. Lord Rosse,

however, had already succeeded in making specula 3 feet in diameter, and the important part to be played in the future by the English amateur was already clearly indicated.

The decade 1840–1850, described by Professor Sampson, is notable chiefly for the discovery of Neptune and the controversy it provoked. The reticence of Adams, who had determined a preliminary place for Neptune in 1843; the inaction of Airy, who never succeeded in explaining his attitude toward Adams; and the inefficiency of Challis, who saw the new planet in one of his sweeps, noted its disk, and failed to identify it as Neptune, will strongly impress the reader of this curious episode in the history of astronomy. The Society was stirred to its depths, but ultimately contented itself by giving twelve testimonials to as many astronomers, omitting the award of the gold medal.

During this decade the development of observatories was rapid, especially in the United States, where the United States Naval Observatory, the Cincinnati Observatory, and the Harvard Observatory with its 15-inch refractor were established in quick succession. W. C. Bond, simultaneously with Lassell, discovered the eighth satellite of Saturn, and was elected the first American Associate of the Society. Other instrumental advances of this period included the electric chronograph (which originated in the United States Coast Survey), Airy's 8-inch transit circle and reflex zenith tube, and Lord Rosse's 6-foot reflector, with which the first of the spiral nebulae definitely recognized as such was discovered in 1845. Baily and Airy first noted and drew the pink "protuberances" at the total solar eclipse of 1842, and Herschel, overlooking Schwabe's discovery of the sun-spot period in 1844, called in 1847 for a continuous daily record of the phenomena of the sun's surface, and advised the Society to organize what might have become a Solar Research Union.

The progress of the Society and the state of physical science during the transition period of the next decade are admirably sketched by the late Colonel Grove-Hills, whose recent death is so deeply deplored. The effect of the classic *Experimental Researches* of Faraday, the rise of the new school of mathematical physics, and the establishment of the conservation of energy and the second law of thermodynamics foreshadowed the birth of astrophysics and the study of stellar evolution. On the observational side this was rendered possible by the development of the spectroscope and the application of photography to astronomy: first, at the total solar eclipse of 1851; next, to the sun and moon, and then (by G. P. Bond at Harvard) to the stars. The publication in 1859

of Kirchhoff and Bunsen's researches on the chemical constitution of the sun and Darwin's *Origin of Species* revealed the possibilities and fixed the direction of the broad study of stellar evolution now to be ushered in.

Professor Newall, though greatly tempted to devote himself to the fascinating steps in the advance of astrophysics during the decade 1860–1870, gives a very complete account of the work and personalities of the Society, and at the same time brings before us an attractive picture of research progress. It was a stirring epoch, in which astronomical photography was first extensively employed; the spectroscope was applied to the determination of the chemical composition, classification, and radial motion of the stars, the analysis of nebulae, and the observation of solar prominences without an eclipse; the daily record of the positions and motions of sun-spots, long since begun visually, was permanently organized by photographic means; while great advances were made in the study of comets and meteors, the measurement of the solar parallax, planetary and lunar theory, and in many other directions. In England, Airy and Adams were still the most conspicuous figures, while De la Rue, Carrington, Huggins, and Lockyer were the leading pioneers in the new and prolific field of astrophysics. In all of this work the Society was a most stimulating influence, serving as a focus of discussion for both professional and amateur astronomers, and providing in the *Memoirs* and *Monthly Notices* the means of printing their papers, which through a liberal system of exchanges were at once placed in the hands of men of science in all parts of the world.

Unfortunately the next decade (1870–1880), as presented by Mr. Hollis with many interesting quotations from the records, was marked in the Society by the repeated clash of vigorous personalities. Proctor's attack on the Astronomer Royal regarding methods of observing the "Transit of Venus"; strong advocacy of government assistance for astrophysical research, weakened by sharp differences of opinion as to the best method of giving it; the belligerency of prominent Fellows, who opposed (referring to solar physics) "the granting of public money for scientific research in cases where it does not appear that results useful to the public will be obtained"; acrimonious debates over the relative merits of candidates for the gold medal; Sadler's attack on the double-star measures of Admiral Smyth—such were the controversies that agitated this tempestuous time. Yet during it the Society acquired its admirable headquarters in Burlington House and its able assistant secretary, Mr. Wesley; published its valuable *Eclipse Volume*, *Reference*

Catalogue of Double Stars, Knobel's *Chronology of Star Catalogues*, and other important memoirs, as well as ten volumes of the *Monthly Notices;* organized jointly with the Royal Society the solar eclipse expedition of 1870; gave necessary financial aid to Sir David Gill's expedition in 1872 to the island of the Ascension for determining the solar parallax by heliometer observations of Mars; and aided in the initiation of the Committee on Solar Physics, which made possible the establishment of the Solar Physics Observatory at South Kensington, now merged in the Solar Physics Observatory of the University of Cambridge. The membership rose to six hundred, and the valuable library, augmented by a rich addition comprising the original sun-spot records of Chevallier, Carrington (1853–1871), Howlett (1869–1876), and the lengthy series of Schwabe (1825–1867), numbered about eight thousand volumes at the close of the decade.

In view of the undesirability of commenting on the parts played by living Fellows, Dr. Dreyer felt that the history of the Society during the period 1880–1920 could not be written in detail. He therefore confines himself chiefly to an interesting survey of the acts of the Council and the internal history of the Society and its publications.

The value of photography to astronomy was enormously enhanced by the advent of the gelatine-emulsion process in the seventies. The great increase in sensitiveness thus realized permitted Henry Draper to photograph the Orion Nebula in 1881, and soon afterward Common successfully entered the field of nebular photography. Gould's photographs of some seventy southern star clusters led Common to propose a plan of photographing the whole heavens, but Gould was unable to join in it. However, Gill's photographs of the great comet of 1882 and the work of the Henry brothers at Paris led to the organization of the International Astrographic Chart, in the preparations for which the Royal Astronomical Society had an important part, while the Greenwich and Oxford Observatories undertook and soon completed their zones.

Other activities of the Society include the establishment of a permanent Photographic Committee, which places on sale astronomical photographs selected from the best anywhere obtainable; the organization in conjunction with the Royal Society of the Joint Solar Eclipse Committee, which has arranged for the observation of many eclipses; co-operation with the Admiralty in the improvement of the *Nautical Almanac;* efforts to encourage solar research in Australia, where a solar observatory has now been established; improvements in lunar nomenclature; the promotion of important projects in geodesy; the establishment of a Geophysical Com-

mittee, which has arranged several meetings for the discussion of geophysical subjects; and effective assistance in the organization of the International Astronomical Union.

Looking back over the century, one is impressed by the great part played by the Royal Astronomical Society and its members in advancing research. Astrophysics has risen from timid beginnings to a position of unlimited possibilities. At first chiefly qualitative, it now employs methods that are unsurpassed in variety, in refinement, and in precision of measurement. As in other fields of research, the British amateur, distinguished by such pioneers as the two Herschels, Baily, Carrington, De la Rue, Lassell, Common, Huggins, and Lockyer, has accounted for an extraordinary share of progress. Half of the presidents of the Society have been amateurs, and half of its British gold medalists also belong to this class. The encouragement thus afforded to those who work with limited instrumental means for the pure love of science, and the possibilities of progress that lie in their hands, could not be better illustrated. Independence of thought; freedom, when desired, from the fixed instrumental demands of routine observation, so necessary in the development of new methods; the intense spur of an interest too keen to be balked by common obstacles—such possessions often outweigh the instrumental advantages of those professional astronomers who have been dulled or hampered by formal requirements or by too intensive routine. In recognizing without discrimination both professionals and amateurs, the Royal Astronomical Society has greatly aided the development of astronomy. Its facilities for publication, regarded by Dr. Dreyer as its greatest service to science, have given us a vast number of papers of the highest class. Its rich library, open with equal generosity to home and foreign members, has made possible many important researches. Its meetings have served as a common forum, where new advances made in all parts of the world have been presented and discussed. Its famous dining club has welcomed many a foreign guest, not only in times of peace but also—as the writer has grateful reason to remember—no less cordially in the uncertain days of war. A society so cosmopolitan in its interests, generous in its benefits, and effective in its efforts to advance science will always serve as a stimulating example, and it is fortunate that its history has been rendered accessible through the joint labors of the able authors of this book.

G. E. H.